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Transfer of Training and Reconstruction of Experience

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I. THE TRADITIONAL VIEW OF TRANSFER OF TRAINING

TRANSFER of training, like democracy, intelligence, learning, curriculum, and the like, has been defined in a variety of ways. Without taking much space to review these definitions it is safe to say that all of them agree on one general pattern. Thus we may define transfer of training as that process of using or applying previously acquired information, habit, skill, attitude, or ideal in dealing with a relatively new or novel situation. So defined transfer of training is a fact which both experience and experiment have amply demonstrated, Thorndike to the contrary notwithstanding. That it is also an ideal we have no reason to doubt so long as we have faith in education and so long further as we subscribe to the view that experience, properly acquired and used, is the best teacher. That it is intimately tied up with our concept of democracy cannot be ignored once we agree that democracy, from one view point, is a faith in the average man's ability to use and modify his experience in order to direct the course of his subsequent behavior. Thus, denial of transfer of training means denial of the verdict of experience, of the efficacy of education, and of the soundness and validity of democracy as a guiding ideal. Furthermore, to deny transfer is equivalent to denying to man that quality called intelligence, which separates him from the non-living and the lower forms of life. In final analysis, education, learning, democracy, and the like terms, all have their root on the assumption that transfer of training is a fact.

This very agreement on the meaning of transfer of training is often responsible for our slovenliness in thinking about it. It makes us oblivious to the fact that the amount, in fact, whether or not transfer takes place at all, is conditioned primarily by the way the information, habit, skill, attitude, or ideal has been acquired. We have assumed, directly or by implication, that there is a one-to-one correspondence between the specific items of the situation and the specific elements of the response. Hence we have only to acquire specific responses and transfer may well be taken for granted. We have had faith in education without the corresponding initiative and insight to bring about transfer in fact. The result, as everybody knows, is a most pathetic example of the faith in the things hoped for and the evidence of the things not seen. Education, if it takes place at all, results in spite of the schools.

This quantitative or mechanistic interpretation of transfer of training is in-

herent in the *doctrine of identical elements* (Thorndike) as it is vaguely assumed in the *theory of generalization* (Judd). The difference in these two theories is one of range and medium of transfer and not in the nature or kind of what is transferred and in the meaning of transfer. This view is responsible for the traditional definition of education as preparation for future living and for the inevitable perpetuation of the *status quo*. Education has become a process of promoting conformity to the requirements of the existing order and hence the subordination of the individual to it. The paradox is that the schools have accepted the democratic ideal and used undemocratic methods to achieve it.

It so happens, fortunately or unfortunately, that the assumption of one-to-one correspondence between the requirements of the situation and the elements that are necessary to meet these requirements, is not true in the type of society such as we have. Progress, like education, takes place in spite of the schools. If this be the case, no two situations are ever the same, and strangely enough, no two items of knowledge, habit, skill, ideal, or attitude are ever the same. The recurrence of any of these in an exact mathematical fashion is a bed-time story.

From these considerations we may therefore conclude that the traditional view of transfer of training is as untenable in fact as it is undemocratic in theory. Hence, our rejection of the application theory of transfer of training, whether it be conceived of as automatic (theory of identical elements) or as consciously generalized (theory of generalization), in favor of a theory that takes into account both the *verdict of experience* and the *requirements of the democratic ideal* which the schools in this county are expected to promote in the pupils.

II. Transfer of Training as Reconstruction of Experience

That transfer of training is a fact which takes place in varying amounts and in manifold ways, depending upon the native ability of the individual and the extent to which favorable conditions for transfer are provided, is a verdict of nearly two hundred studies that have been made on the subject from 1890 to 1935.[1] If so, how then does it take place other than in a way that is required by the one-to-one correspondence theory which is referred to above? If no two situations and no two elements of response are ever the same, how then does transfer take place, in fact, how is learning possible at all? This dilemma is not resolved by the ingenius device of denying transfer altogether (Thorndike) or by the more subtle assumption that the mind has the innate capacity to generalize experience (Judd), but by the more arduous though more effective process of re-interpretation of the meaning of transfer of training and a corresponding change in the method of bringing about transfer in fact.

This re-interpretation should take its clue from the changing and dynamic nature of the social order and the flexible character of the human organism. There seems to be no need for elaborating on the proposition that we are living in a rapidly changing society, even though an educational leader (H. C. Morrison) has the temerity to declare that all this "talk about society changing rapidly is an immense amount of poppycock." In like manner, we can well assume without fear of successful contradiction, that flexibility is characteristic *par excellence* of the human organism. In fact, paradoxically enough, the human organism is so flexible that it can be made rigid and fixed if we control the conditions appropriately. Thus, we can *train* the individual in such a way that he behaves like an automatom, or we can *educate* him so that he can act intelligently.

Thus it may be safely concluded that

[1] P. T. Orata: "*Transfer of Training and Educational Pseudo-Science.*" *Educational Administration and Supervision*, 21: 241–64 (1935); also *The Mathematics Teacher* (May, 1935).

whereas the dynamic nature of the social order makes transfer of training necessary, the flexible character of the individual organism makes it possible. If at the end there is to be an effective correspondence between the situation and the response so as to bring about a stable adjustment, both the situation and the response must be modified simultaneously during the process of their active interaction. It is this simultaneous reconstruction of the situation and the response in order to bring about effective equilibrium that constitutes transfer of training and, at the same time, provides a clue as to the method of bringing it about. This in effect is what we mean by acting intelligently. To use Dewey's famous definition, transfer (which in this case is synonymous with education) is that process of modifying or reconstructing experience that will enable the individual to deal more effectively with subsequent experience.

III. Experience, Continuity, Reconstruction

It should be apparent from what has been said just now that transfer of training, like freedom, democracy, learning, or intelligence, is not wholly or even largely a gift but an achievement or a conquest. It is the result or effect of the simultaneous reconstruction of the situation and the response that matters in transfer of training and not the presence of some hypothetical elements that are said to be identical. The problem of transfer of training disappears when identity is achieved. To put it somewhat differently, the identical elements are the *result* and not the cause of transfer of training. These identical elements are not however just facts, processes or data in nature or experience, that await notice (Thorndike) or discovery through generalization and application (Judd), unless the identical element is the person himself.

The problem of providing for transfer is not therefore a mere matter of discovery of identical or corresponding elements, important though this process is; it is not just a matter of analysis of varying concomitants like the biologist discovering the principle of evolution or the physicist discovering the law of gravitation, although this process may well enter in. It is rather a problem of creation, construction, building up of attitudes, ideals, beliefs, and practices in the light of some desired social value. It is more of a problem of *human engineering*, to borrow a technological term, than that of discovering the weight and velocity of an electron. Instead of a Davenport, we should have a Luther Burbank; in place of a Faraday, we should have a Thomas Edison. Not that Davenports and Faradays are not important, but as *educators* as contrasted with men of science, our job is not to discover laws but to make men. We build character, we do not find it. The child learns to be honest, not by discovering that honesty is the best policy, but by so reacting to situations and the situations reacting back that results in the making over of both the situation and the response. The situation acquires a new meaning because of the consequences of reaction, and this new meaning so guides the reaction that if differs from what it was.

To be more specific, a pupil learns his mathematics not by discovering the answers or even by proving them,[2] but by making his calculations in terms of the requirements of the problem, by analyzing the parts and putting them together again in different patterns in order to arrive at a satisfactory solution. The child does not just discover that 22 plus 0 makes 22. He sees this combination and "adds" to it his previous experience with 2 plus 0 and in so doing he transforms it into something that he can deal with satisfactorily. It is not just a matter of reading 2 plus 0 and 22 plus 0 although that may well enter into

[2] Distinction should be made between real learning and learning that is merely verbal. The test is the amount and certainty of transfer to life situations.

the situation, but rather the realization of the principle that zero added to something, whether it be a number or a pound of flesh, does not change its value. Without this principle, which is a meaning and a creation rather than an entity or an abstraction, the pupil may well continue dealing with zero combinations without noticing except accidentally, the *identical elements*. But with this principle, the child can deal with other combinations involving zero without the ordinarily large amount of practice that will otherwise be necessary to learn them. Furthermore, and this is even more important than transfer in the field of mathematics, the child if properly educated to deal with his experience, will likely learn that sand cannot be transformed into gold, that social security, old-age pension of two hundred dollars a month, relief of unemployment, and the like, cannot be attained and balance the budget and reduce the rate of taxation at the same time. But notice that there are nowhere any sembiance of identical elements, but an extension of the imagination will enable one to make use of the principle (in a modified form) that zero added to something does not increase its value, or even more glaring still, a negative number added to a positive number reduces rather than increases the value of the number.

The value of such a generalization later in dealing with equations in algebra or with differential and integral calculus cannot be overestimated. By a proper approach the child can be so taught the simple number combinations that in later years he will be able to use his early experiences in dealing with higher processes in mathematics, as well as with experiences or problems in other aspects of his life. We must however, realize that transfer in any of these cases is never automatic except accidentally. In the teaching of the higher processes and in dealing with other experiences, the child should be so taught that he learns to make the application by the process of making over the new situation or stating it in such a form that the old principle, if correspondingly modified, will work. We have many instances in algebra and higher mathematics where an expression which is otherwise complex may be transformed so that it becomes simplified, i.e., reduced into a form that makes the application of specific formula or principle previously learned possible and relatively easy. A problem that is otherwise complex and difficult to deal with may be reduced to a simple equation by proper classification and interpretation of the parts and their meanings. A very complicated arithmetical problem when reduced to an algebraic form becomes simple and easy. And we have all heard of the phrase "God eternally geometrizes." Life in some form is mathematical, and so is progress. Even love can be expressed in terms of an equation if we look at it as a perfect balance and harmony of the parts that compose it. As a matter of fact, many people, Americans especially, have so learned their mathematics that progress, success, happiness, standard of living, and the like are understood and appreciated largely, if not wholly, in terms of quantitative measures which are mathematical. The case of the scout master who asked the guide of the Dresden museum "how much" the Sistine Madonna was worth is only one of the many examples of the American traveler giving himself away. The present tendency in American education of quantification, mechanization, standardization, and objectification represents the application of mathematics to the professions. This transfer or application is at its maximum in the behavioristic interpretation of the mechanism of learning and of life. Hence the dogma, "Everythings that exists, exists in some amount and can be measured."

As was said a moment ago, transfer is never automatic except accidentally. For maximum transfer to take place from one situation to another there should be at least three processes to be provided,

namely, *experience*, *continuity*, and *reconstruction*. These are all active processes not passive and abstract contemplations. The child learns that 2 plus 0 is 2 not by merely being told about it but by actually observing concrete situations representing this combination. Likewise he sees that adding weight to one part of the balance disturbs the equilibrium unless an equivalent weight is added to the other side. In short, he learns a specific fact or item of knowledge by experiencing,—by seeing, touching, hearing, testing, and smelling, or by comparing or relating it with previously experienced situations.

But separate items of experience, knowledge, habit, attitude, or ideal do not transfer in and of themselves, except in rare instances, unless some way is provided whereby the individual perceives the relation of these to previous or subsequent items of experience. In short, there has to be *continuity* of experience to render experience in one situation efficacious in enabling one to deal more satisfactorily with subsequent situations. Thus, it is necessary for a child to have learned the fundamental processes of addition, subtraction, multiplication, and division before he can deal with the higher processes of compound interests, logarithms, and differential equations. The child in the elementary grades can deal with mixed numbers by rote, but he can deal with them more easily if he has previously learned how to add similar fractions, how to find the least common multiple, and how to reduce dissimilar fractions to similar fractions. This is what we mean by the principle of learning, "proceed from the simple to the complex." The equivalence of this in this discussion is, "provide continuity of experience if transfer is to be expected."

Again, there may be concrete experience and continuity of experience without substantial transfer because in the process of learning and teaching *reconstruction* has not been achieved. The child goes through a process by routine fashion, by following a formula. The problem is stated so clearly that all he needs is to make the application. He does not learn how to analyze the problem by isolating the relevant from the irrelevant, by proper classification of the factors according to the requirements of the problem, and he fails to see the need of modifying the principles or formulae that he might apply. Mathematical problems are usually so classified and stated that the child does nothing about it except to apply the formula or process that is being dealt with. In other words, the child is not taught how to think independently. An extreme, though not an unusual example of this is a textbook in a second grade arithmetic which contained several problems to be solved by the pupils. The problems were so classified that each group was preceded by a caption such as *Division by Five*. All the problems under that group contained processes that should be dealt with in the way indicated so that all the child needed to know was the caption and the skill of dividing by 5. This is equivalent to saying that the child fails to learn to think because the textbook writer and the teacher do all the thinking for him.

IV. Transfer of Training and Current Educational Movements

These three requirements of transfer of training, *experience*, *continuity*, and *reconstruction*, may well be the basis of critically evaluating the current movements in education, namely, traditionalism, scientific education, and a few of the "fifty seven" varieties of progressive education. Each of these movements succeeds or fails, according to this viewpoint, in proportion as it meets or fails to meet these requirements. In fact, these criteria may well be used as a measuring device to evaluate any educational system that is provided.

Traditional education has its virtue in attempting to provide *continuity* through the logical sequence or organization of subject matter, but fails to promote trans-

fer, first, because it encourages verbal learning, and second, it does not encourage and cultivate reflective thinking in the pupils. The most conspicuous instance of this is the field of mathematics where from the first grade to the graduate school of the university the individual learns abstract symbols and processes with a minimum of meaning and application to life situations. Consequently from a practical point of view the only value of mathematics except in the case of the simple processes is in the study of higher mathematics. Provision for *verbal* continuity and reconstruction does not result in transfer.

Scientific education attempted to rescue education from verbal learning through proper selection of situations that have meaning and need to the child and through teaching only the processes that are needed in dealing with these situations. Thus, if it is found by actual analysis of life situations that the fractions, 7/8, 5/14 and the like are used infrequently or not at all, they need not be taught in the schools. To do so would be a waste of time and money. Again, if by application of scientific technique processes can be classified with reference to their difficulty, the proper sequence of teaching these processes is according to their degree of difficulty. The easier ones should be taught first regardless of their logical connections. Also, since all learning is specific, only those things that are necessary for adult living should be taught. Having learned these the child is equipped for life. It should be at once apparent that this type of education does not aim to provide continuity of experience, and thereby makes reconstruction of experience unnecessary. In fact, it is this type of education that denies that transfer of training exists as a fact, and is futile as an ideal for which to strive. The dogma of this school of educational theory is "Learn what you learn. It does not transfer." Hence the emphasis on specific objectives in curriculum planning and on the doctrine of habit formation in learning and method. Thinking, and therefore transfer, is provided by the expedient process of doing away with it altogether.

Progressive education, speaking of the *extreme left* variety, like scientific education, attempts to vitalize child experiencing without in the least being concerned let alone perturbed by the need for continuity and reconstruction of experience. This school of educational theory makes experience the be-all and end-all of life and of education. The present is the concern of the school. Maximum experiencing in the present is the best preparation for the future. Directly or by implication, the proponents of this movement have no use for transfer of training. In fact, they tend to identify it with formal discipline which is, by assumption, untenable. If experiencing is the only need of the child, continuity and reconstruction are definitely out of the picture.

Progressive Education, speaking of the variety called *education by indoctrination*, has all the three elements of experiencing, continuity, and reconstruction. Problems of living are introduced in the schools, a frame of reference is provided that furnishes a basis both of organization and reconstruction of experience. But we must not forget that transfer and reconstruction are not enough. Russia and Germany have these to a maximum degree *within the framework* that is provided in advance by the powers that be. In effect, this school of thought is effective in providing for transfer of training, but it is also undemocratic in the extreme. In fact, it is a case of educating for democracy by undemocratic methods. The ideal is sacrificed for the attainment of the process.

Progressive education of the kind that attempts to foster *independent reconstruction* is similar to the type called education by indoctrination except in one respect, namely, that it does not provide a frame of reference in advance for the child. In fact, it is its aim to make it possible for each child to define and achieve a frame of reference for himself without fear or

inhibition because of social pressure, taboos, and standards of personal conduct that are provided for in advance for him. Instead of reconstruction *within* a framework, this theory advocates reconstruction *of* the frame work. This type of education makes personal and social problems central in the educative process, places a premium on the reconstruction of conflicting patterns, and promotes independent thinking as the method of democracy. So conceived and realized this type of education is calculated to promote maximum transfer of training as well as lead to the achievement of the democratic ideal. It is this type of education that can be justified in a social order that accepts democracy as its guiding ideal.

V. Specific Applications to the Teaching of Mathematics

If education, according to the last point of view, is to foster the cultivation of thinking that is both critical and independent, then whatever happens in the classroom and whatever subject is taught should help promote this ideal. Fortunately, mathematics has always been regarded as a subject that lends itself best both as a mood of thinking and as a tool for thinking. However, no one, even the most ardent advocate of mathematics, would support the view that mathematics as it is now organized and taught promotes thinking in the pupils that will transfer to fields other than mathematics. In fact, many studies fail to convince us that there is a significant carry-over or application of learning from one field of mathematics to any other. It can even be said that the objectives of these subjects in terms of specific learnings and skills are far from being realized in the ordinary school. That is to say, the pupils pass algebra without knowing how to solve problems that involve algebraic processes, or are promoted from the second to the third grade without knowing how to add, subtract, or even count.[3] To expect transfer under these conditions is equivalent to reaching for the moon. Where there is none to transfer,—well it is a case of adding zero to a pound of flesh.

It remains to add that the kind of training that will transfer to the social situation is not obtained, except rarely, by even the most effective study of formal subjects. We cannot ignore or even be indifferent to the social scene and at the same time expect the learner to be able to deal with it satisfactorily through the medium of his training in school. It is possible so to train the child that he can see the meaning of 2 plus 0 in the most complicated mathematical problem without being able in the least to see the corresponding social fact that we cannot get something for nothing, radio advertisers and political humbugs to the contrary notwithstanding. To put it differently, the various subjects of the curriculum, most of all mathematics, cannot be expected to result in automatic transfer to the social situation that confronts the child, unless, by proper instruction and organization of the school, these subjects can be made a "way of life" and be so regarded and used by the child himself. This is what we mean by "humanizing education" in the concrete. It is the kind of teaching that will promote intelligent and functional learning.

Granting that this be the case, how precisely should the teacher go about reorganizing mathematics so that he will be able to achieve the social objectives that are indicated?

1. He should conceive of mathematics both as a *mode of thinking* and as a *tool for thinking*. As a tool for thinking mathematics differs essentially from other fields of knowledge since it is primarily quantitative, precise, and objective. But as a mode of thinking it differs in no essential respect from physics, history, language, or even metaphysics. All thinking as a method involves the definition of the problem, the formulation and testing of hypotheses, and verification of results; it

[3] W. A. Brownell, et al.: "Types, Characteristics, and Problems of Learning." *Rev. Edu. Res.*, Vol. III, No. 4, pp. 289–94.

T. H. Briggs: *The Great Investment;* Harvard University Press, 1930, pp. 124 ff.

involves certain assumptions or meanings that control the interpretation and solution of the problem; it implies certain attitudes such as, suspended judgment, precision and accuracy within the limitations of the data that may be available; it is experimental and not dogmatic in its procedure; and above all, it is individual through and through since in no way may one be justified in saying that one person can think for another.

2. The subject of mathematics should be brought into relation with the other subjects of the curriculum in such a way as to permit the organization of the school program in terms of the child's daily life problems and activities. Negatively speaking this means the elimination of the boundary lines that now separate the various subjects. Any problem no matter how simple involves concepts of numbers, of relations, of quantity; it also involves consideration of values or preferences; it requires experience, which may be mathematical, purely informational, historical, physiological, psychological, and the like; it involves trying this approach and then that until the correct solution is found. In fact, the problem may be so simple as to require just a moment of reflection, or it may be so complicated that it requires an entire life time to even understand let alone solve it. In any case the whole of human experience is brought to bear on any problem that may confront the individual.

This means that the problems and activities of boys and girls and not the logical requirements of the various subjects become the bases of the organization of the school program. We do not teach in order that the pupils might learn the subject, but we should so teach or so provide opportunities to the pupils that they can learn in school how to think, how to live together, how to solve their immediate problems—in short, how to act intelligently. It is this, acting intelligently, that transfers and not mere knowledge of subject matter.

3. While this will mean the abolition of mathematics as a special subject except for those who are preparing for specific professions that require advanced mathematics,[4] it does not necessarily preclude the specific learning of number concepts, combinations, relations, and other processes that are ordinarily regarded as belonging to the field of mathematics. One has to learn certain necessary computations with facility and accuracy; to understand and interpret certain mathematical concepts and technical vocabulary; to read simple graphs, charts, tables, diagrams, and other types of materials in which quantitative data are arranged in a systematic, orderly manner. However, the treatment of these specific processes should take place as they are needed in the solution of the problems which the pupils consider vital and important. The practice of "learn first and apply afterward" should be modified so as to read: "learn as you apply." This does not preclude the need for drill on processes that are difficult, but the drill should take place only as it is needed to enable the pupils to deal with a problem that requires the ability to use these processes. It may be necessary to take time off, so to speak, and this should vary with the difficulty of the process. It may well be that one week or even one month will be needed for the pupils to master the process sufficiently to be able to use it. Why not? *The difference in this case is that while in the ordinary method the pupils do not often see the reason why they are learning the addition of dissimilar fractions, in this new approach they are motivated in learning it by the fact that they see the reason for doing it.*

4. While the mathematical processes are to be taught as they are needed in the solution of a specific problem, provision should be made in the organization of what is learned so that it will transfer in a way that will enable the individual to deal with other problems as well as to learn related processes. This means provision for continuity and sequential learning, which as we have tried to show, is a primary factor in promoting reconstruction of experience. This is where logical organization is to be provided. This process of relating present experience with past or future experience facilitates as well as vitalizes learning. The habit when established enables the individual to utilize his background of information, habits, skills, and attitudes in dealing with new situations. Otherwise, such background becomes isolated possession which, except rarely, does not function in directing the course of subsequent experience. It is one thing to say that learning should not proceed from a logically organized

[4] Even here there is ground for the belief that mathematics should be taught in relation to the problems of the specific professions, i.e., engineering.

subject-matter which is the practice of traditional education; it is an entirely different matter to say that we should promote the logical organization of experience. In this latter case logical organization is an end point toward which to strive and not the beginning of instruction. It is another name for interpretation of experience. Experience that is not in the end organized is a hodgepodge and does not, except rarely, transfer the claims of some progressive educators to the contrary notwithstanding.

5. The practice of simplifying problems so as to present only the mathematical side of it and neglecting the other aspects which are likely as important if not more so, is, at best, falsification of facts and, at worst, ineffective in providing for transfer to life situations. There is never any problem that is purely and simply mathematical, problems in engineering included. If there is transfer at all it is likely to be of the undesirable type, such for instance, as regarding progress only in terms of measurable quantities, viz., increase in salary, rise in the value of commodities, variety in the courses offered, increase per capita earning, or even increase in divorce rates. The height of willful disregard of other considerations than those that are mathematical in nature is reached in the conception of education known as *scientific. When Thorndike pronounced the thesis "Everything that exists, exists in some amount, and can be measured," and then proceeded to build up a system of measurement of educational results on that basis, he inaugurated the most vicious attempt to quantify, mechanize, and standardize the educative process.*

6. Emphasis should be placed as much if not more in the process of thinking through the problem than in the correct answer. The getting of the correct answer as the goal is responsible for many faulty and undesirable habits among the pupils as shown in an analysis of the causes of difficulties in problem-solving. For instance, Banting found the following errors of pupils:[5]

1. Failure to comprehend the problem in whole or in part because, in part, of teaching, of carelessness in reading. His desire to get the answer quickly or his lack of care and attention results in either total error or impractical solution.
2. Lack of ability to identify the proper process or processes with the situations indicated in the problem. One may understand the processes very well and yet not know which to choose to solve a particular concrete problem. The lack of this ability, not to know whether to add, subtract, multiply, or divide in a concrete case is characteristic of so-called dull pupils in arithmetic, and is the chief cause of the painful stabbing, the mere juggling of figures that is the despair of the teacher in the middle and upper grades.
3. The habit of focusing the attention upon the numbers and being guided by them instead of by the conditions of the problem. For example, in a problem in which the essential fact was that a man received $18 for 24 hours work, a number of children divided 24 by 18 to find the hourly wage, because 18 is a smaller number than 24. Another example is the fact that many children feel that they must use every number stated in a problem in its solution. (This last tendency is in itself an evidence of the operation of transfer and habit which is built up by repetition of solving simplified problems.)
4. Akin to the foregoing, pupils are sometimes completely nonplussed by large numbers. Though they can read the problem and identify the process when smaller numbers are used, they are perplexed by numbers larger than those of every day experience, which should not be given in the first place.
5. The habit of being guided by some verbal sign instead of making an analysis of the problem. For example, in the problem from Buckingham's test, "A boy has 210 marbles, and lost one third of them. How many had he left?" A bright boy whose answer was 70, explained his mistake as follows: "Miss ——— (his teacher) told us that if we saw 'of' after a fraction in a problem we were to multiply."
6. The failure to recognize the mathematical similarity to type problems which the pupils understand, because of some unusual situation in the problem in question. For example, the pupil who readily solved problems dealing with the purchase and sale of familiar things, failed when given a problem dealing with the purchase and sale of a farm.[6]

[5] G. O. Banting: *Second Yearbook of the Department of Elementary School Principals*, pp. 411–21. (See Brueckner and Melby: *Diagnostic and Remedial Teaching*, pp. 230–32, for a summary of the article.)

[6] The above summary of the relevant parts of Banting's article appears in Brueckner's and Melby's *Diagnostic and Remedial Teaching*, pp. 230–32.

7. Proper and valid instruments or measures of evaluation of results that are anticipated should be invented. Nearly every test, standardized or new type test, in mathematics has to do with the determination of facility and accuracy of computation and problem solving of the most formal type. In short, only the computational parts of mathematics are evaluated, and little or nothing is done to measure the effects, if any, of mathematics taught as a mode or method of thinking. Such aspects as the definition of the problem, the formulation of hypothesis, the testing of hypothesis, the verification of results, interpretation of data, critical evaluation of assumptions, the presence of such attitudes as suspended judgment, precision and accuracy in accordance with the limitations of the data, independent judgment, and the like, are never measured. (Exception should be made in the attempt of the Progressive Education Association Evaluation of the Eight Year Study Committee under Dr. R. W. Tyler to construct instruments to measure some of these so-called intangible results of teaching. The test by Hartung on "Interpretation of Data" is suggestive of further leads. Fawcett of the University High School at Columbus, Ohio, is making a study on the problem and is obtaining some fruitful results. His article on "Teaching for Transfer" in the December, 1935, issue of MATHEMATICS TEACHER is worth reading.)

It must be said here that measurement of these products can be so formalized (the study of Hartung referred to above is an example of formalizing and standardizing thinking in mathematics) that they become as useless in evaluating, to say nothing of promoting thinking, that will transfer to life situations in and outside the school. Unless the problem is vital to the pupils, and unless the problem is unsimplified, we can continue testing these higher processes without promoting them one iota. The pupils as of old will be more interested in the correct answer than in the logical validity and soundness of the process of solution which they follow.

8. There should be proper and constant diagnosis of pupils' difficulties not only in computation but also, and perhaps more so, in problem solving. This diagnosis should become the basis of remedial work which may well be individual and intensive. Where specific drill is deemed necessary it must be used even to the consternation and irritation of the so-called progressive educators. Drill in this case becomes vital to the pupil since he knows that he has difficulty in specific processes the mastery of which is essential in dealing with concrete problems that interest him. This diagnosis can be as specifically and concretely scientific as the rigor of mathematics can provide, but the teacher should make sure that the so-called intangible results receive their due share of analysis. Remedial work which follows should be as specific as the diagnosis, and again, emphasis should be placed on the processes involved in "mathematics as a mode of thinking" to which we have already referred.

If the above recommendations are too revolutionary for the teacher in the standard school it might be expedient at the present time to adopt the recommendations made by Dr. Judd in many of his writings, most particularly in his *The Psychology of Social Institutions*, namely, to socialize the school subjects. This may be a half-way measure, but as a transition it has its outstanding merits. After all, the practice in the last three thousand years of teaching subjects as separate compartmentalized entities cannot be made over even in a generation. Dr. Judd recommends, and rightly so, that the pupil from the kindergarten up should be led progressively to realize the part that mathematics has played in the development of civilization and what it has yet to play in the further advancement of progress in industry, commerce, science, and the arts. More concretely, such courses as might be given in any field of mathematics, emphasize the history of mathematics in its relation to the development of science and technology, and should make proper application of the concepts learned and developed to the immediate problems of individual and group living of the boys and girls in school. More specifically the following steps seem to be necessary in order to implement this transitional stage of improving the teaching and organization of mathematics instruction:

1. Creation by the teacher and pupils working together of concrete and practical situations within the level of development of the pupils, leading or giving rise to the need of specific mathematical skills, information, and facility.
2. The introduction of the mathematical concepts, combinations or processes in a form that will enable the pupils to generalize their learning experience.
3. The application of the generalizations made and of the skills learned to specific and concrete situations. This and the preceding step will facilitate transfer of training from mathematics to other fields of mathematics and to life.
4. All along as occasion demands, the pupils should be led to appreciate the value of mathematics to life in making progress possible in the past and in guiding the destiny of the future.

VI. Concluding Statement: Democracy a Method, Not a Doctrine or Creed

If democracy is a method, not a doctrine or creed, it must be some specific kind of method. It cannot be a method of external authority dictating conformity to its requirements under penalty of suffering; not the method of custom, or working or thinking by precedents of the past; not the method of routine, or persisting automatically, without asking for a reason, in paths worn deep, smooth, and easy by long repetition; not the method self-interest of individuals or of a class dressed up to look like public service; not the method of trial by force to see which is the stronger in cannon or gunpowder, or in command of money and credit.

It should be a method of "free intelligence" through free inquiry and independent judgment on the intricate problems that confront us every day. When a civilization like ours with its conflicts, contradictions, and serious maladjustments awaits the strong hand and brain of its citizens, we cannot afford to waste time by teaching and learning isolated subjects in the guise of scholarship and academic freedom. Rather we should dedicate ourselves to enabling each person to prepare himself for the arduous task of exercising his God-given prerogative of freedom to think independently, feeling unafraid due to freedom from ordinary inhibitions of fear of authority, custom, habit, or group pressure from varied sources. Viewed in this light education becomes a vital preparation for living because it is the whole of living, and consequently, mathematics becomes converted into a tool and mode of thinking as valuable as it is indispensable because it frees the individual from the limitations of routine, habit, prejudices, and physical weakness. So conceived learned, and taught, mathematics becomes a "way of life," a safeguard of freedom of thinking, a tool and a method that will help humanize education. This is the concrete meaning of the thesis "transfer of training through reconstruction of experience."

What This Country Needs

When failure is mentioned we become sentimental and think too little about the social waste that will ensue if we pamper the individual in his irresponsible practices. . . . What would society have lost if Pasteur, who failed, or if Einstein, who failed, had been coddled and passed? . . . When I reflect on the history of civilization and on the problems of present-day society, it seems to me that there was never a time when students should be held more rigorously to high standards, never a time when students needed more to be taught that understanding can be acquired only by mastering systematic knowledge. . . . I make a special plea for education that puts lime in the bone, iron in the blood, and organized knowledge in the minds of the youth of this generation.—L. D. Coffman, in *The Educational Record*.

A Few Observations on Robert Recorde and His "Ground of Arts"

By Emeline R. Ebert
Mentor Junior Senior High School, Mentor, Ohio

The Ground of Arts: Teaching the perfect work and practice of Arithmetick both in whole Numbers and Fractions, after a more easie and exact form then in former time hath been set forth:
Made by M. Robert Record, D. in Physick.
Afterward, augmented by M. John Dee.
And since enlarged . . .
by John Mellis.
And now diligently perused, corrected, illustrated and enlarged.
London.
1646.

According to the historian Ball, Robert Recorde was born in Tenby in Pembrokeshire *c.* 1510 and died in London in 1558. He was entered at Oxford and obtained a fellowship at All Saints College in 1531. Later he migrated to Cambridge where in 1545 he took a degree in medicine. He finally settled in London and became physician to Edward VI and Mary Tudor. At the time of his death he was confined to the King's Bench prison, probably for political reasons. His volume on arithmetic, first published in 1540 or 1542, represents the work of a young man in his early thirties.

The work is in dialogue form. At the beginning, we read

Master—. . . So that of number, this may I justly say, it is the onley thing almost that separateth man from beasts. He therefore that shall contemn number, declareth himself as brutish as a beast, and unworthy to be counted in the fellowship of men. But I trust there is no man so foule over-seen, though many right smally do it regard.

This brief summary leaves little room for us to doubt the Master's conviction of the importance of his subject. His next step is to justify the method which he employs in the text, and again he speaks plainly and convincingly, for he feels that he is right.

Master—. . . because I judge that to be the easiest way of instruction, when the Scholar may ask every doubt orderly, and the Master may answer to his question plainly . . . so there will be some that will finde fault, because I write in a Dialogue: but as I conjecture those shall be such as do not, cannot, or will not perceive the reason of right teaching, and therefore are unmeet to be answered unto, for such men with no reason will bee satisfied . . . Therefore (Gentle Reader) though this book can be but small aid to the learned sort, yet unto the simple ignorant (which needeth most help) it may be a good furtherance and mean unto knowledge.

We note that it is the Scholar who launches the dialogue.

Scholar—Sir, such is your authority in mine estimation, that I am content to consent to your saying, and to receive it as truth, though I see none other reason that doth lead me thereunto: whereas else in mine own conceit it appeareth but vain to bestow any time privately in learning of that thing, that every childe may, and doth learn at all times and hours, when he doth anything himself alone, and much more when he talketh or reasoneth with others.

Master—Lo, this is the fashion and chance of all them that seek to defend their blinde ignorance, that when they think they have made strong reason for themselves, then have they proved quite contrary. For if numbring be so common (as you grant it to be) that no man can do anything alone, and much lesse talk or bargain with other, but he shall still have to do with number: this proveth not number to be contemptible and vile, but rather right excellent and of high reputation, sith it is ground of all mens affairs, in that without it no tale can be told, no communication without it can be continued, no bargaining without it can be duely ended, or no businesse that man hath, justly completed. These commodities, if there were none other, are sufficient to approve the worthinesse of number. But there are other innumerable, farre passing all these, which declare number to exceed all praise. Wherefore in all great

works are Clerks so much desired? Wherefore are Auditors so richly fed? What causeth Geometricians so highly to be enhaunced? Why are Astronomers so greatly advanced? Because that by number such things they finde, which else would farre excell mans minde.

This is an example of motivation par excellence. The episode which follows is a striking example of bringing the general statements above within the focus of the pupil's own vision with a homely illustration.

Scholar—Verily, Sir, if it bee so, that these men by numbring their cunning do attain, at whose great works most men do wonder, then I see well I was much deceived, and numbring is a more cunning thing then I took it to be.

Master—If number were so vile a thing as you did esteem it, then need it not to be used so much in mens communication. Exclude number, and answer to this question: How many years old are you?

Scholar—Mum.

Master—How many dayes in a weeke? How many weeks in a year? What lands hath your Father? How many men doth hee keep? How long is it since you came from him to me?

Scholar—Mum.

Master—So that if number want, you answer all by Mummes: How many miles to London?

Scholar—A poak full of plums.

Master—Why, thus you may see, what rule number beareth, and that if number bee lacking it maketh men dumb, so that to most questions they must answer Mum.

Scholar—This is the cause, Sir, that I judged it so vile, because it is so common in talking every while: Nor plenty is not dainty as the common saying is.

Master—No, nor store is no sore, perceive you this? The more common that the thing is, being needfully required, the better is the thing, and the more to be desired. But in numbring, as some of it is light and plain, so the most part is difficult, and not so easie to attain. The easier part serveth all men in common, and the other requireth some learning. Wherefore as without numbring a man can do almost nothing, so with the help of it, you may attain to all things.

Here is a teacher who is not afraid to use a piece of meaningful nonsense when the point in question is so precisely struck. We feel sure that since this Master is to create his own Scholar, the latter will be a real one, presenting the real problems that the mere mechanical mouthpiece Recorde might have invented would so easily evade.

The Master goes on to show the bearing and effect of "Arithmetik" on "musick, physick, law, grammer, philosophie, divinity, and armies." His remark on the legal aspect, if made in more modern times, would be certain to bring forth a torrent of editorial comment and judicial indignation.

Master—This often times causeth right to bee hindered, when the Judge either delighteth not to hear of a matter that hee perceiveth not, or cannot judge for lack of understanding; this cometh by ignorance of Arithmetick . . . If I should (I say) particularly repeat all such commodities of the noble Science of Arithmetik, it were enough to make a very great book.

At this point one catches a suppressed sigh on the part of the Scholar who probably is afraid that the Master will allow himself to be so carried away with the point he desires to make that he actually will repeat all the commodities of the noble science. The young fellow loses no time in saying,

No, no Sir, you shall not need: For I doubt not, but this, that you have said, were enough to perswade any man to think this Art to be right excellent and good, and so necessary for man, that (as I think now) so much as a man lacketh of it, so much hee lacketh of his sense and wit.

Not one word has the Master uttered concerning the content of that which he intends to teach his Scholar, and it is the latter who takes the initiative in seeking the knowledge of his own accord. The Master's talk has registered the desired effect, but it was not empty propaganda, as is emphasized by the bit of dialogue which follows.

Scholar—I beseech you, Sir . . . and if yee will bee so good as to utter at this time this excellent treasure, so that I may be somewhat inriched thereby, if ever I shall be able, I will requite your pain.

Master—I am very glad of your request, and will do it speedily, sith that to learn it you bee so ready.

Scholar—And I to your authority my wit so subdue, whatsoever you say, I take it for true.

Master—That is too much, and meet for no man to bee beleeved in all things, without shewing of reason. Though I might of my Scholar some credence require, yet except I shew reason, I do it not desire. But now sith you are so earnestly set this Art to attaine, best it is to omit no time, lest some other passion coole this great heat, and then you leave off before you see the end.

Scholar—Though many there be so unconstant of minde, that flitter and turn with every winde, which often begin, and never come to the end, I am none of this sort, as I trust you partly know. For by my good will what I once begin, till I have it fully ended, I will never blin.

Master—Better it were never to assay, then to shrink and flie in the midway.

The questioning mind is sought instead of blind acceptance of fact, and how this Scholar utilizes the opportunity to question that which the Master presents to him! The former is not such a backward lad and gives evidence of knowing a few principles of elementary psychology himself. Since he is a puppet-pupil, for whom Master Recorde pulls the strings, it is quite evident that the Master *knew* his pupils, their reactions, questions, and tendencies.

In the first actual discourse which deals with numeration the Master presents a brief outline of the material he intends to cover and then attacks the initial topic, numeration. He explains the two values a number may have as ". . . One alwayes certain, that it signifieth properly, which it hath of his form; and the other uncertain, which he taketh of his place . . . A place is called the seat or room that a figure standeth in."

Master— . . . when it is in the first place, though many other do follow: as for example: This figure 9, is ix, standing alone.

Scholar—How is he alone and standeth in the middle of so many letters?

Master—The letters are none of his fellows. For if you were in France in the middle of a thousand Frenchmen, if there were no Englishmen with you, you would reckon yourself to be alone.

In the teaching of place value the Master has his Scholar prick with his pen above the fourth, seventh, and each succeeding third place in order to more easily distinguish the values, in the same way that commas make the distinction clear to us today. In pricking the monstrosity, 230864089105340, the Scholar is puzzled by the zero, and when the Master straightens out the difficulty for him, the lad comments, "Then in the example above, I have pricked well enough: for though the Cypher that is pricked signifie nothing, yet must he have the prick, because hee came in the thirteenth place."

In the beginning work on the fundamental processes, subtraction is defined as ". . . nothing else but an Art to withdraw and abate one summe from another, that the remainer may appear."

Scholar—What do you call the Remainer?

Master—That you may perceive by the name.

Scholar—So me thinketh: but yet, it is good to ask the truth of all such things, lest in trusting to mine own conjecture, I be deceived.

Master—So it is the surest way. And, as I see cause, I will still declare things unto you so plainly, that you shall not need to doubt. Howbeit, if I do overpasse it sometimes (as the manner of men is to forget the small knowledge of them to whom they speak) then do you put me in remembrance yourselfe, and that way is surest.

This last remark is reminiscent of Professor Clifford B. Upton of Teachers College, Columbia University, who has often told his beginning algebra classes that if each student will allow no explanation to pass in class without understanding it or without asking for further explanation if he does not clearly understand it, he will guarantee that every student in the class will achieve success in the new subject.

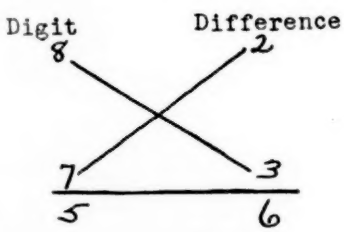
The Master's usual practice here and throughout the book is to present the process by giving the general rule, followed by an example which either he or the student works in application. The Scholar then works others alone, often discovering while he works some new

angle or twist of the original rule. This is precisely what is attempted today when we try to direct out teaching to supply the needs of the pupils as these needs become apparent. So the Scholar who has been subtracting readily is suddenly confronted with the problem of taking a larger number from a smaller number and must turn to the Master for assistance. The Master proceeds to give him his reply which is a form of the additive method of subtraction which has become so favored in the modern elementary school.

Multiplication is defined by Master Recorde as ". . . an operation whereby two sums produce a third: which third sum so many times shall contain the first, as there are Unites in the second. And it serveth instead of many Additions." When the need for multiplying "greater digits" arises, the Master presents the following method in his own way.

Master—And as for the small digits under 5 it were but folly to teach any rule, seeing they are so easie, that every childe can do it: but for the multiplication of the larger digits, thus shall you do.

First, set your digits one right over the other, then from the uppermost downwards, and from the nethermost upwards, draw straight lines so that they make a crosse, commonly called Saint Andrews Crosse, as you see here. Then look how many each of them lacketh of 10, and write that against each of them at the end of the lines, and that is called the difference: as if I would know how many are 7 times 8, I must write those digits thus.



Then doe I looke how much 8 doth differ from 10, and I find it to be 2, that 2 doe I write at the right hand of 8, at the end of the line, thus.

After that I take a difference of 7 likewise from 10, that is 3, and I write that at the right side of 7, as you see in this example.

Then do I draw a line under them as in Addition, thus.

Last of all I multiply the two differences, saying 2 times 3 make 6, that I must ever set under the differences, beneath the line: then must I take one of the differences (which I will, for all is like) from the other digit (not from his owne) as the lines of the crosse warne me, and that that is left, must I write under the digits . . . and then there appeareth the multiplication of 7 times 8 to be 56.

For ease and surety in working the Master makes a table for his Scholar to assist him with the multiplication of the "greater summes." After the necessary preliminary explanation and example by the Master, the Scholar does a problem of his own making.

Scholar—If there bee no more to bee observed in it, then can I do it, I suppose, as by this example I shall prove. There is a piece of ground which containeth 1365 yards in length, and 236 yards in breadth: I would know how many yards square there is in all this piece of ground:

My own question at this point is whether a Scholar, who is learning the fundamentals of multiplication, will be familiar with the method of obtaining an area and be able to use the square units understandingly as this young man undoubtedly does. I am aware that he is an exceptional fellow, but I have my doubts about this particular point, unless some satisfactory explanation can be discovered. It may be that the over-zealous Master has merely tripped up on a minor point or that he is intentionally giving the reader the illustration of the area formula. If the latter were true, it would appear that his goal might have been attained more easily if he had made a special point of presenting it to the Scholar instead of allowing the latter to spring it on us voluntarily. It seems to be the multiplication process which is being stressed and not the area rule mpl ed

As each of the digits in the multiplier is used as an individual multiplier, "he" is immediately given a "fine dash with my pen," and before long the Scholar, non-

chalantly explaining his examples, remarks, "Then I give 3 his dash," and in another instance, ". . . and dash him slightly with my pen." Later the Master uses the term "cancel," the root of so many evils, according to us today.

Master—I would know how many dayes it is since the Nativity of our Lord and Savior Jesus Christ, unto this year 1645. Which to performe, you must multiply this present year 1645, by the dayes in the whole year, which are, 365.

This example the diligent fellow carries on to hours and even minutes of his own accord!

Scholar—This is marvellous, mee thinke, that such great matters may so easily bee achieved by this Art, which heretofore I ever thought had been impossible, as infinite sorts of people are of that minde.

Master—Truth it is that knowledge hath no greater enemy then ignorance, for this is one of the least of ten thousand things that may be done by this Art.

The check of casting out nines is given for multiplication until the Scholar is familiar with division, which, when it does arrive shortly, is of the "scratch" variety. A few short cuts for multiplying and later for dividing by 10, 100, 1000 etc. are given here, and when the Scholar asks whether "there is no other forme of division in practise but this," the Master replies, "Yes verily, there are other formes in practise, but because I love brevity I will declare onely one which I first learned of, and is practised by that worthy Mathematician, my ancient and especiall loving friend, Master Henry Bridges, wherein not any one figure is defaced or cancelled. As if I should divide 72 by 6, first place them thus 6)72. Then if you please you may write the divisor in a loose paper that it may more easily without cancelling or defacing of the work be applied to, and removed from the dividend at pleasure." With the tool of division well sharpened, the following bit of discourse closes the subject.

Scholar—Truly Sir, these excellent conclusions do wonderfully more and more make mee in love with the Art.

Master—It is an Art, that the farther you travell, the more you thirst to go on forward. Such a Fountain, that the more you draw, the more it springs: and to speak absolutely, in a word (excepting the study of Divinity, which is the Salvation of our soules) there is no study in the world comparable to this, for delight in wonderfull and godly exercise: For the skill hereof is well knowne immediately to have flowed from the wisedom of God, into the heart of Man, whom he hath created the chiefe image and instrument of his praise and glory.

Scholar—The desire of knowledge doth greatly incourage me to bee studious herein: and therefore I pray you cease not to instruct me further in the use hereof.

The "feat of Reduction" from "summes of grosse denomination" to "summes of a more subtile denomination, and contrariwise" follows, and a glance at the tables of values of "most usuall Gold-coyns throughout Christendome, and what they are worth of currant English money this present year 1630" makes one long to bargain in "coyns" with such delightful and charming names. Any trading would savor of romance and adventure, in my estimation, if counted in "spur Royalls, Thistle Crowns, Duckets of Aragon or Carolus Gilden." Exchange was so uncertain and coins so numerous that the Master gave his young Scholar only a few of the "currant coynes" and their values, as he proceeds to the problem of weights.

Master—After a Statute made Anno 11 H. 7. there ought to be but one sort of weight. As 24 Barley corns dry, and taken out of the middest of the Ear, do make a pennyweight, 20 of those penny weights make an ounce; and 12 ounces a pound of Troy weight, by which is weighed Bread, Gold, Silver, Pearle, Silke, and *such like*. But commonly there is used another weight called Haberdupoise; in which 16 ounces make a pound.

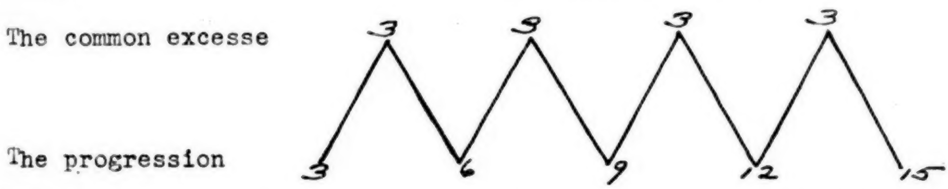
When our young sophisticates today groan at the tables to which they are occasionally requested to refer, it might be fitting to have them glance at this particular chapter of the Ground of Arts. They might then be able to count their blessings with great fervor. Even with the

limited facilities of his day Recorde had a goodly store of tables, charts, examples, and diagrams in his text. In beginning arithmetical progression he resorts to the jigsaw device here pictured.

After a practice period on the several propositions relating to proportions there is an incident which I found amusing.

Scholar—Sir, I pray you heare mee frame one example more.

Master—I am well pleased so that yee bee short, for you make me more longer here then willingly I would have been: but I cannot perceive how I could have omitted anything as yet, without your great lack thereof. (He may fear a little "stalling," but also justifies his own tardiness.)



Scholar—If I had received 85 pounds of certaine men, but of how many I have forgotten, yet I remember that the first gave mee 7 pound and the last 27 pound, and every payment after other did rise by a like summe. And the man for whom I received this mony conditioned with mee, that of every payment I should have twelve pence for my labour: now unlesse I can by Art finde the truth of the case I am like to lose the most part of my reward.

Master—I perceive you can handsomely frame an example which should concerne your owne gaine: I pray you let mee see how you would do justice in this point.

Scholar—I adde the first and last together, that maketh 34; by which I divide 85, thus: Why how now? Sir, here is a remnant of 17, in which 34 cannot be had: so that now I am in the briers for doubling of my quotient, and farewell then both my Justice and a good lump of my gains.

So the Master adroitly manipulated his ill-fated Scholar into the "briers," in order to illustrate the point of a remainder which must be doubled, a performance new to the Scholar. It so happens that the remainder in this case will reduce to one half, making the process of doubling simple, but the point has been well dramatized as the Scholar leads himself into the wilderness and is forced to cry out for aid.

In pursuing "Progression Geometricall," one of the first problems the Master gives is "a question of Satten."

A mercer hath 12 yards of Satten, which he valueth at 16 shillings the yard, and selleth the same 12 yards to another man to be paid as followeth: That is to wit, for the first yard to have one shilling, for the second yard two shillings, for the third yard foure shillings, for the fourth yard 8 shillings etc., doubling each number following, till the twelfth and last yard. The question is, who hath made the better bargaine of the buyer or the seller.

The Master applies the principle which he has already explained, and this conversation closes the question.

Master— . . . and so much hath the Mercer for his twelve yards of Satten: which is 17 pound, 1 shilling, 3 pence a yard. But I thinke you will buy none so deare.

Scholar—No, Sir, by the grace of God this yeare.

There follows the "question of an horse."

Master—Then what say you to this question? If I sold unto you an horse having 4 shoes, and in every shoe 6 nayles, with this condition, that you shall pay for the first nayle one ob: for the second nayle two ob: for the third nayle foure ob: and so forth, doubling untill the end of all the nayles. Now I ask you, how much would the price of the horse come unto?

Another is "a question of bricks."

Master—A Lord delivereth to a bricklayer a certaine number of loads of Bricke, wherof he willed him to make twelve walles of such sort, that the first wall should receive two thirdels of the whole number, and the second two thirdels of that which was left and so every other, two thirdels of that that remained: and so did the Bricklayer: and when the 12 walls were made, there remaineth one load of Bricke. Now I ask you, how many load went to each wall, and how many load was in the whole?

To this the dumbfounded Scholar replies, "Why Sir, it is impossible for me to

tell." The Master solves the problem quite easily, and the chapter closes with the following remarks, which make it quite evident that the "furtherance" of the subject is due to the Scholar's request, which, of course, is prompted by the excellent presentation and motivation of the Master, who has not as yet even reached into the second layer of "tricks" in his bag of the latter.

Scholar—Lo now it appeareth easie enough. Now surely I see that Arithmetick is a right excellent Art.

Master—You will say so when you know more of the use of it: For this is nothing in comparison to the other points that may be wrought by it.

Scholar—Then I beseech you, cease not to instruct mee further in this wonderfull cunning.

In attacking the next topic the Master exercises the right of the teacher, to change the order of presentation, allowing the extraction of roots to "passe" for awhile, and teaching instead the "feat of the rule of Proportion," which for his Excellency is called "The Golden Rule": whose use is by three numbers knowne, to find out any other unknowne; which you desire to know as thus. If you pay for your board for three months sixteen shillings, how much shall you pay for eight months."

The Master uses the device of "a crooked draught of lines," such as I have pictured and stresses very pointedly the necessity of numbers of like denomination being grouped together.

Master—Other diversities there be of working by this rule, but I had rather that you would learne this one well, then at the beginning to trouble your minde with many formes of working, sith this way can do as much as all other, and hereafter you shall learne the other, more conveniently.

This may have been the instigation of the movement, which has been slow to reach us, namely to simplify the beginning work and to make it so thorough that the real value will not become lost in a maze of complications for the learner.

The Golden Rule or Rule of Proportion Backward or Reverse, sometimes called the "backer or reverse rule of three," follows, and the Master's initial point is to distinguish carefully between its purpose and that of the preceding one.

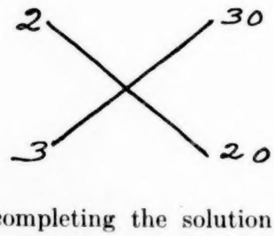
Master—But in this rule there is a contrary order, as this: That the greater the third summe is above the first, the lesser the fourth summe is beneath the second; and this rule therefore you may call the Backer or Reverse Rule, as in the example.

I have bought 30 yards of cloth of two yards breadth, and would have Canvas of three yards broad to line it withall, how many yards should I need?

Scholar—Why, there is none so broad.

Master—I doe not care for that, I doe put this example onely for your easie understanding for if I should put the example in other measures, it would be harder to understand. But now to the matter. . . .

This young Scholar would certainly win the hearts of the present day exponents of the "genuine problem," as he challenges the conditions of the problem. The Master's reply is probably the first defense of the artificial problem. It is a neat piece of justification and clinches the assertion that Robert Recorde was a modern master who studied his methods and his pupils and was in all probability centuries ahead of his generation.



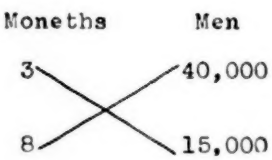
After completing the solution as the diagram here illustrates, the Master concludes with, " and now because ye found fault with my Example, how say you, perceive this?" The Scholar responds with a somewhat dubious, "Yes Sir, I suppose." The Master may sense the unconvinced

state of mind of his Scholar for he speaks for a time of the bearing of Arithmetick on practical affairs

Master—This rule is so profitable for all estates of men, that for this rule onely (if there were no more but it) all men were bound highly to esteeme Arithmetick.

By this Rule, may a Captaine in War, work many things, as Master Digges in his Stratiocos doth declare: Onely now in this my simple addition, for a taste and incouragement. I wil enlarge the Author with a question or two more, wishing you and every my Countreymen, or Gentlemen whatsoever, that by nature bee anything given to Military affaires, to bee familiar and acquainted with this Excellent Art, the which hee shall finde not onely at the Sea, but also in the Campe and Field-service, aboundantly to aid him, either in Fortification, paying of the Souldiers wages, charges of Ordnance, Powder, Shot, Munitions and Instruments whatsoever, as for example.

If it should chance a Captaine which hath 40,000 Souldiers to be inclosed with his Enemy, that he could have no fresh purveyance of victuals, and that the victuals which hee had would serve that Army but onely three moneths, how many men should hee dismisse to make the victuals to suffice the residue eight moneths?



Was ever a subject more surging in its appeal to growing youth than that of war and army life? Master Recorde might be subjected to an investigation for indoctrinating his pupil with militaristic motives if he had used such material in certain places today, but in his age he was comparatively safe. If I interpret this problem correctly, I am at a loss to understand why the Scholar does not challenge the situation by demanding to know how a Captaine "inclosed with enemy" could dismisse men. Where would the "dismissees" go but into the arms of the enemy? The Scholar was undoubtedly subdued by the reception which his former challenge received and may have decided to allow this point to "passe."

I would like to quote just one passage from the Golden Rule Double (or the Double Rule of Proportion direct).

Master—If a Captaine over a band of men did set 300 pioners aworke, which in eight hours did cast a trench of 200 Rods: I demand how many labourers will be able with a like trench in three houres, to intrench a Campe of 3400 Rods.

Scholar—I thinke I am now in the Backehouse ditch: for I know not well which way to go about it. And besides that, truly I thinke I shall never come to preferment that way, my growth is so small.

Master—You know not how God may raise you hereafter by knowledge and service into the favour of your Prince, for the availe of your Countrey.

Example for Navigation: Sir Francis Drake, a man greatly honoured for his knowledge, was not the tallest man, and yet hath made as great an adventure for the honour of his Prince and Countrey, as ever Englishman did.

Scholar—Sir, I thanke you for your goode incouragement. My minde, though I be little, is as desirous of knowledge, as any other: I have pondered now a little of it, and thus I set forth the worke . . .

After the Master has divulged the Rule of Fellowship and its working, the Scholar makes the following observation, "Sir, I have attentively beheld you working, and the more we travell herein, the more me thinke I am in love with this excellent Art."

A complete explanation of the method of computing with counters, generously illustrated with diagrams, showing the relative positions of the counters in various processes, occupies the succeeding chapter. It must have been a sore trial to the author and the printer to accomplish this feat with the facilities and the equipment of that time.

Master—Now that you have learned the kinde of Arithmetick with the Pen, you shall see the same Art in Counters: which feat doth not onely serve for them that cannot read and write, but also for them that can do both; but have not at some time their pen or tables ready with them.

Let me quote from the dialogue at the close of this section.

Scholar—If that be all, you shall not need to repeat again that which was sufficiently taught already: and except you will teach me any other feat, here may you make an end of this Art, I suppose.

Master—So will I do as touching whole number, and as for broken number I will not trouble your wit with it, till you have practiced this so well, that you be full perfect, so that you need not to doubt in any point that I have taught you, and then may I boldly instruct you in the Art of Fractions or broken Numbers: wherein I will also shew you the reasons of all that you have now learned.

. . . .

Scholar—Yet in the mean season, I cannot stay my most earnest desire, but importunely crave of you some brief preparation toward the use of Fractions, whereby at the least I may be able perfectly to understand the common works of them, and the vulgar use of those rules, which without them cannot well be wrought.

Master—If my leasure were as great as my will is good, you should not need to use any importunate craving, for the attaining of that thing, whereby I may be perswaded that I shall any wayes profit the Common-wealth, or help the honest studies of any good members in the same: wherefore while mine attendance will permit me to walk and talk, I am willing to help you as I may.

In introducing fractions, as well as at other times, the Master asks the Scholar for his conception of the new tool and then proceeds to build up or tear down from this, his own idea in his own manner. I would rate the Scholar extremely high for his rapid assimilation of the "broken number" concept. His quick acceptance and subsequent use of the new instrument is almost uncanny. It would appear that even the Master has his doubts, for after the young fellow gives a summary of the "severall varieties" of fractions, he says, "You have said well, if you understand well your owne words," To this remark the Scholar responds, "If it shall please you, I will by an example in the parts of an old English Angel, expresse my meaning."

The varieties of reduction overwhelm the poor student who remarks that "this distinction in Doctrine delighteth me much, but more with hope than present fruit: for as yet I do not understand scarcely the varieties, and much lesse the practice and use of their works." Later he proves that after all he is a human young man by observing and remarking quite earnestly, "Sir, I heare your words, but I doe not understand many of them."

When the Master is "shewing" a short cut of reduction by "casting away Cyphers" from the numerator and the denominator, the Scholar attempts an example in which he obtains 4/65 by reducing 400/650. When his error is pointed out by the Master, the lad is quick to perceive and to reply, "I confesse my fault, which came from too much haste. I was more gladder of the Rule then wise in using it."

In multiplication of fractions Master Recorde is careful to explain the size of the result obtained when two fractions are multiplied as compared with the size of the individual fractions. The Scholar is interested and seeks an explanation which is thorough and convincing when given. Division the Master ties up with proportion, stating that "$\frac{1}{2}$ divided by $\frac{1}{4}$, maketh 2, which must be sounded not 2, but twice, declaring that $\frac{1}{4}$ is containeth twice in $\frac{1}{2}$."

When somewhat later the Master gives a corrected table of the weights of the farthing white loaf for the different prices of a quarter of wheat, he justifies his action with this statement.

Master—These two Tables I have set severall, because no man should thinke that I would either adde or take away from any Law those parts which might of right seeme either superfluous, either diminute: but yet I may not bee so curious as to neglect manifest errours, which is not onely my part, but every good Subjects duty with sobriety to correct. And for avoiding of offence, I have rather done it in this Private Book, then in any Book of the Statutes itself, trusting that all Men will take it in good part.

Scholar—I would wish so, but I dare not so hope, sith never good man that would reforme errour, could escape the venemous tongues of envious detractors, which because they either cannot, or like not to doe any good themselves, do delight to bark at the doings of other, but I

beseech you to stay nothing for their perverse behavior.

Master Recorde "causeth" his pupil to become rather advanced in his observations with this last comment, which seems somewhat out of order for the Scholar, who might have commented on the act, but not in this way which is simply the Master using the Scholar as his mouthpiece.

When the Rule of Fellowship is being considered, one of the problems given is the following.

> Foure men got a booty, or prize in time of war, the prize is in value of money, 8190 pound, and because the men bee not of like degree, therefore their shares may not be equall! but the chieftest person will have of the booty the third part, and the tenth part over: the second will have a quarter and a tenth part over: the third will have the sixth part: and so there is left for the fourth man a very small portion but such is his lot (whether he be pleased or wroth) hee must bee content with one 20 part of the prey. Now I demand of you, what shall every man have to his share?

This is too much for the Scholar who is quite frank to admit that "you must bee faine to answer to your owne question else it is not like to be answered at this time."

The Master introduces Alligation.

> *Master*—Now will I goe in hand with the Rule of Alligation; which hath his name for that by it there are divers parcels of sundry prices & sundry quantities, alligate, bound, or mixed together: Whereby also it may well be called the Rule of Mixture; and it hath great use in composition of Medicines, and also in Mixtures of Metals; and some use it hath in Mixtures of Wines: but I wish it were lesse used therein then it is now adayes.
>
> There are foure sorts of wine of severall prices, one of 6 pence a Gallon, another of 8 pence, the third of 11 pence, and the fourth of 15 pence the Gallon. Of all these Wines would I have a mixture made to the summe of fifty gallons, and so the price of each gallon may be 9 pence. Now demand I, how much must be taken of every sort of Wine?

At this point the Master presents a problem concerning a Mintmaster in which the word Karect is used for the first time and brings forth no comment from the Scholar. This so astonishes the Master that he is forced to ask, "But how chanced you made no doubt of that new name Karect?"

"Because I thought it out of time to demand such questions now, seeing you make such haste to end," is the Scholar's nonchalant reply!

Introducing the famous "Rule of Falshood," Master Recorde defines it, and then proceeds to deal with it at great length.

> *Master*—The Rule of Falshood, which beareth his name not for that it teacheth any fraud or falshood, but that by false numbers taken at all adventures it teacheth how to finde those true numbers you seeke for . . .
>
> I sometimes being merry with my friends, and talking of such questions, do call unto them such children or idiots, as hapned to be in the place, and so take their answer, declaring that I would make them solve those questions, that seemeth so doubtful . . . which Rule for readiness of remembrance, I have comprised in the few verses following, in forme of an obscure Riddle.
>
> Ghesse at this work as hap doth lead,
> By chance to truth you may proceed,
> And first work by the question,
> Although no truth therein be done.
> Such falshood is so good a ground,
> That truth by it shall soon be found,
> From many bate too many moe,
> From too few take too few also:
> With too much joyn too few again:
> To too few adde too many plain.
> In cross wise multiply contrary kinde,
> And all truth by falshood for to finde.
>
> The sense of these Verses, and the summe of this Rule is this.
>
> When any question is proponed appertaining to this Rule, first imagine any number that you list, which you shall name the first position, and put it in stead of the true number, and then worke with it as the question importeth: and if you have missed, then is the last number of that worke either too great or too little: that shall you note as hereafter shall be taught you, and you shall call it the first errour.
>
> Then begin againe, and take another number, which shall bee called the second position and worke by the question: if you have missed againe, note the excesse or default as it is, and call that the second errour. Then multiply crosse-wise the first position by the second errour, and againe the second position by the first errour, and note their totalls: Then marke

whether the two errours were both alike, that is to say, both too much, or both too little: or whether they be unlike, that is one too much, and the other too little: for if they bee like, then shall you subtract the one totall from the other (I meane the lesser from the greater) and the remainer shall be your Dividend: so must you abate the lesser errour out of the greater, and the residue shall be the Divisor. Now divide the Dividend by that Divisor, and the quotient will shew you the true number that you seeke for.

But, and the errours be unlike, then must you adde both those totalls (which you noted) together, and take that whole number for the dividend, so shall you adde both errours together and that whole number shall be the Divisor, and the quotient of that Division shall give you the true number that the question seeketh for, and this is the whole Rule, with this signe —, betokening too little.

And because you shall no more forget this part of the Rule, take this briefe remembrance.

Unlike require Addition.
And like desire Subtraction.

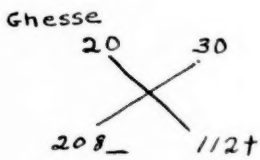
. . . .

There is a servant that hath bought of Velvet and Damask for his Master 40 yards, the Velvet at 20 shillings a yard and the Damask at twelve shillings, and when hee commeth home, his Master demandeth of him, how much he hath bought of each sort: I cannot tell (saith hee) exactly: but this I know that I paid for the Damask 48 shillings more than I paid for the Velvet: now must you ghesse how many yards there is of each sort.

Scholar—Although the ghesse seemeth difficult, yet I will prove what I can doe: for I remember your saying, that it forceth not how fond or false the ghesse bee, for it bee somewhat to the question, and not an answer of a contrary matter.

Therefore first I imagine that he bought 20 yards of Damask, for which hee should pay after the former price 240 shillings: then must hee needs have of Velvet other 20 yards (to make up the 40 yards) and that would cost 400 shillings. So that the totall of the price of the Damask is lesse then the summe paid for Velvet 160 shillings, and should be more by 48. Therefore the first errour is 208, too little. Then begin I againe, and suppose he bought of Damask 30 yards, that cost 360 shillings, then had hee but 10 yards of Velvet, which cost 200 shillings: and now the price of the Damask is greater then the price of the Velvet by 160 shillings, and should bee but 48, therefore is the second errour 112 too much, which I set in forme of figure as here doth appear.

Then do I multiply in crosse wayes 208 by 30, and the summe will be 6240. Also I multiply 112 by 20, and there will amount 2240. And in as much as the signes of the errours be unlike, I know I must worke by Addition, therefore adde I these two totalls together, and they make 8480, which is the Dividend: then adde I also the two errours together, 208, and 112, and they make 320, which is the Divisor: wherefore dividing 8480 by 320, the quotient will be $26\frac{1}{2}$, which is the true summe of yards of Damask that he bought, and in Velvet 13 yards $\frac{1}{2}$.



Master—There are three men that do owe money to me, and I have forgotten what the totall summe is, and what the particulars be.

Scholar—Why then it is impossible to know the debt.

Master—Peace, you are too hasty, there is more helpe in it then yet you see, I have three severall notes whereby it appeareth that I did conferre their debts together, and found the debt of the first and second to amount to 47 pound, and the debt of the first man and the third did make 71 pound, and the second man his debt with the third did rise to 88 pound. Now can you tell what every man did owe, and what was the whole sum?

The Scholar very naturally guesses, but again naturally places guesses for each of the three amounts, a mistake which is commonly make by one who is attempting the trick for the first rime. The Master soon sets him on the right path, and he goes on to a sucessful solution.

Master Recorde brings in the historical allusion from time to time, and I wish that time and space permitted the retelling of his version of the well known tale of Archimedes. I must beg off by using the Master's own excuse, "If my leasure were as great as my will is good . . ."

When the work is brought to a completion the Scholar says to his teacher, "Sir, although I cannot recompense your goodness, yet I shall alwayes doe mine endeavor to occasion you not to repent your benefit on me thus employed."

The reply of the Master, "That rec-

ompence is sufficient for your part," brings the book to its close.

Robert Recorde, I believe, approached the pattern set for the teacher-ideal, because he knew and loved his subject so devotedly and believed in its use and influence in practical as well as theoretical bearings.

As a teacher this Master never lost sight of his pupil and moreover never failed to stress the human, practical, and interesting phases of his applications. This was accomplished with direct or indirect means but was always achieved so simply that the learner could with a minimum of effort grasp and retain the real significance of the respective problems as they were presented. No topic was merely "handed out" without a carefully planned and developed preliminary justification and explanation. In the majority of cases by skillful questioning and guidance, the Master leads the Scholar to the point where, in order to proceed, the latter needs the new arithmetical equipment, which is promptly supplied, practiced and applied. This process continues until a further need is apparent.

Master Recorde's developments are interesting. Some proceed from a general rule or principle as he states it with minor variations and new twists which occur as the Scholar uses the rule, and others grow out of the Scholar's own idea of a new subject, corrected, enlarged upon, and drilled on, under the Master's supervision. Sufficient practice in the beginning of new work seemed to be a particular point with this teacher, as was the giving of simple problems until the pupil had achieved a thorough understanding. Robert Recorde would never have sacrificed understanding of the subject at hand on the altar of speed and manipulation, a practice which is too often true today.

No section of this small volume appeared dull or uninteresting to me, and this was probably due to the constant reference to the "vulgar" and homely applications of the subject and the Master's willingness to digress for a bit of humor or whimsical philosophy at any convenient time. This fits in very snugly with the highly advertised and widely discussed, although often misused, "socialized" procedure of modern teaching, in which there is supposed to exist between teacher and class a spirit of friendliness and cooperation as the learning process goes on its way. This Scholar gives every evidence of having the greatest respect for his Master, but this respect is never couched in the fear and awe which might so easily have ruined all achievement. The Master enjoys his Scholar and what he is able to give him from his vast store of knowledge, and at the same time, the Scholar enjoys his Master and is spurred on to acquire all the knowledge he can in the time he is allowed.

The Relative Effectiveness of a Large Unit Plan of Supervised Study and the Daily Recitation Method in the Teaching of Algebra and Geometry

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Purpose of Experiment. It was the objective of this investigation to furnish experimental data throwing light on the question of the relative effectiveness of a plan of supervised study and the conventional daily recitation method. In the supervised study procedure, unit assignments of from four to twenty days were employed. One to three days were spent in making the assignment and preparing the class to attack it. This state was then followed by several days of supervised study in which for the greater part silent individual study was employed though occasional class discussion was used. Each unit was concluded with a "recitation" in the form of an oral discussion or an oral or written quiz over the unit.

In the daily recitation plan the students covered the same materials though the assignments were not always the same, the recitation plan group being given daily assignments involving work to be done outside of class. They had no time for supervised study.

The experiments were conducted in two units, one during the first semester of the year and the other during the second semester. Two sections of 19 pupils each were employed in the experiments in algebra and two of 15 each in geometry. All pupils were students in the Teton County High School at Choteau, Montana.

Control of non-experimental factors. The same teacher, Mr. Hunziker, taught both sections in each subject. The class periods were of equal length, 45 minutes each, and care was taken that one section did not suffer more than its mate by reason of interference of assemblies and similar events. Pupils absent were required and enabled to make up the work lost. In two cases the section having the greater total absences was superior in achievement and in the other two the reverse was found. No pupil was included in the experimental data who was absent more than seven days.

Pupils in each section were dropped from experimental consideration and these were so selected as to leave groups thought to be equivalent in ability as judged by the Miller mental test (I.Q. and M.A.) chronological age, and initial scores on tests to be described later which were used to measure progress during the experimental period. In no instance was the difference between the means of the experimental and the control groups greater than one-fourth of the standard error of either group and in the majority of instances the difference was less than 10% of the standard deviation of either group. Likewise the groups were so selected as to be of approximately equal spread in the factors used as judged by approximately equal standard deviations. Approximately equal numbers of boys and of girls respectively were included in the sets of two paired sections.

As a further guarantee of equality of ability between the paired sections, the methods were exchanged at the conclusion of the first semester, the group having been taught the first semester by the

large unit supervised study plan now being taught by the daily recitation plan during the second semester and vice versa.

The measurement of progress. In the algebra sections, progress was measured by the Douglass Standard Survey Algebra Tests and a "new type" test constructed by the instructor, Mr. Hunziker. The coefficients of reliability of these tests were estimated by first computing the coefficient of correlation between final test scores and chance halves of the tests and correcting these coefficients by use of the Spearman-Brown prophecy formula.[1] These coefficients were as easily great enough to permit reliable comparisons between the two sections as far as chance errors of measurement are concerned. They were as follows:

First semester algebra
- Douglass Survey Test I, Form A .78
- Hunziker's new-type test .71

Second semester algebra
- Douglass Survey Test II, Form A .74
- Hunziker's new-type test .86

In geometry, progress was measured by means of an objective test constructed by Mr. Hunziker. The coefficients of reliability of these tests estimated as in the case of the algebra tests were 91 for the first semester test and 92 for the second semester test. The first semester test consisted in the proofs of two originals, twenty statements to be matched, twenty-seven completion exercises, and five constructions. The second semester test consisted in fifty true-false statements, twenty matching items, completion exercises totaling fifty blanks, and twenty numerical problems.

These tests were given at the beginning and at the close of the appropriate semesters and the differences between the initial and the final scores taken as measures of progress or growth.

Relative progress in algebra. During the first semester there was no significant difference in gain as measured by either test between the paired sections in algebra, though slight differences were obtained in favor of the supervised study plan. During the second semester very material and reliable gains were experienced by the daily recitation groups.

TABLE I

Chances in 100 that the Superiority Is in the Direction of the Obtained Superiority

Semester	Test	d	SD_d	$\frac{d}{SD_d}$	Chances in 100	Difference in favor of
First	Douglass Test I	.11	.94	.11	54	Supervised study
	Author's Test I	.74	1.75	.42	66	Supervised study
Second	Douglass Test II	3.78	1.00	3.80	99+	Daily recitation
	Author's Test II	15.67	4.68	3.30	99+	Daily recitation

These facts permit several speculative conclusions among which are the following:

1. That the daily recitation plan is really superior after the beginnings of algebra have been mastered.

2. That it is better to use supervised study the first semester and the daily recitation plan the second semester.

3. That the group of pupils following the large unit plan the first semester and the daily recitation plan the second semester was, in spite of the attempts to equate the two groups, a more capable section, which, with the advantage of having been taught how to study, far outstripped the less able group during the second semester.

4. That the daily recitation method was superior and that the daily recitation group was also superior and did better even when "handicapped" with supervised study.

The data of the experiments when given appropriate analysis revealed no relation-

[1] Given and described in any good book on statistical methods in education.

ship between intelligence of the pupils and superior gain on one method, the coefficients of correlation all being less than .10. Likewise, no significant differences were found in the variabilities of gain of the respective groups.

Table II

Chances in 100 that the Superiority Is in the Direction of the Obtained Superiority

Test	d	SD_d	$\frac{d}{SD_d}$	Chances in 100	Differences in favor of
First Semester Test	3.27	2.93	1.11	86	Daily recitation
Second Semester Test	6.67	6.11	1.09	86	Daily recitation

Relative progress in geometry. In the geometry sections, the results both semesters materially favored the daily recitation group. While in neither case was the difference great enough as compared to the standard error of the difference to insure complete independence from chance errors of sampling and measurement, yet the fact that, even though the groups were interchanged between semesters, there was a natural difference favoring the daily recitation method, is very strong evidence in favor of that plan.

As in the algebra experiments, no relationship was discovered between method used and intelligence of pupil and no significant differences were discovered between the two groups as to variability gains among pupils.

General conclusion. While hard and fast conclusions may not be drawn, there is very convincing evidence that, with pupils trained in the daily recitation method for years and with little experience in the large unit plan of supervised, the daily recitation plan in the hands of at least some teachers of mathematics if not a majority of them, will yield better results as measured by gains on written examinations than the large unit plan of supervised study employed by Mr. Hunziker. While one should not generalize too freely from any one such experiment, however carefully conducted, the data of this study throw some doubt on the superiority of the large unit supervised study-plan of teaching elementary algebra and geometry.

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"It is also highly probable, though not so easily proved, that a child's power of rapid learning has declined more before he enters school than it ever will afterward; and that it steadily declines thereafter, most rapidly in youth, and least rapidly in old age. To learn to speak our own tongue is probably the greatest intellectual feat any one of us ever performs. To master the integral calculus is, comparatively speaking, mere child's play."—Roy Hilton in *Harper's* for December, 1936, page 4.

Mathematical Difficulty in College Physics

By Karl F. Oerlein
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The mathematical nature of physics is well known. Teachers of physics contend that much of their students' difficulties in the course are attributable to poor preparation in mathematics. Indeed, that the success or failure in physics rests to a large degree on the mathematical preparation of the students is no longer a supposition. Bless,[1] Bailey,[2] and others have shown a positive effect between students' mathematical preparation and success in physics.

As to the causes for this difficulty in physics, there appears to be but three reasonable possibilities.

1. The mathematics actually used in the physics course may be more than the students have had.
2. The students may have received credit for sufficient mathematics but they have not mastered it.
3. The mathematical situations encountered in the physics course may be so different from those learned in the mathematics class that students are unable to recognize them without assistance.

The results of some recent studies, made on both the secondary school and college levels, throw light on these possibilities.

Discussing the first possibility: Is more mathematics used in the physics course than the students have had? The results of studies, on the secondary school level, by Reagan,[3] Lemmer,[4] and others, show that the mathematics used was not, in general, more than the students had been exposed to. The result of an extensive study,[5] on the college level, indicates substantially the same conclusion. In this investigation, the stated mathematical requirements for admission to 379 first courses in general college physics from 211 institutions throughout the United States and Canada were compared with the mathematics used in them. It was found that the mathematics implied by the stated requirements was adequate for the work done in the courses. While it was true that the range of the mathematics used in these first courses varied widely, the stated mathematical requirements varied accordingly.

If a teacher uses more mathematics in the course than that implied by the stated prerequisites, the responsibility for student failure rests squarely with that teacher. There is, however, no serious objection to a "stiff" mathematically treated course provided that the prerequisite mathematics has been set accordingly. Hence, guided by the purpose of the course, it is within the duty of the instructor in physics to decide how much mathematics will be used. The mathematical requirements for this course is set accordingly. The instructor has a right, then, to expect that students with credit for this amount of mathematics will be able to use it without too great a strain.

Evidence seems to show that, on the whole, physics teachers have gauged their mathematical needs for their particular courses correctly and have kept within the mathematical limits implied by the mathematics accredited to their students.

This brings up the second possibility: Have students mastered the mathematics

[1] Bless, A. A.: *Science*, LXXV: 343, Mar., 1932.

[2] Bailey, R. C.: *Sch. Sci. and Math.*, XXXV: 89, Jan., 1935.

[3] Reagan, G. W.: *Sch. Sci. and Math.*, XXV: 292, Mar., 1925.

[4] Lemmer, J. G.: *Sch. Sci. and Math.*, XXX: 41, Jan., 1930.

[5] Oerlein, K. F.: *Mathematical Requirements for the First Courses in General College Physics.* Doctor's Thesis, University of Pennsylvania, 1936.

for which they have received credit? Many studies have revealed the relatively low degree of mastery of mathematics. The excellent study by Lueck,[6] on the college level, is typical of the results. In this study, an analysis was made of the arithmetical and algebraic errors of college freshmen pursuing first year physics. The study revealed a surprisingly weak mastery of elementary mathematics.

Doubtless, many mathematics teachers would be among the first to recognize this discouragingly weak mastery of mathematics. Poor mathematical preparation has been blamed, unjustly, for many scholastic ills but it is an injustice to indict mathematics teachers as a whole. They are probably as efficient as any other group of teachers. The criticism is more properly directed at the content of the courses which they teach, the lack of integration of mathematics courses of study, and, most important, lack of coordination with related fields. Mathematics is not an end in itself except for a small group of prospective teachers and professional mathematicians. This is quite true of other subjects, physics not excepted. But, since the learning of physics, for whatever purpose, depends so largely on the mathematical preparation, it is the duty of mathematics teachers to modify their courses so as to conform more nearly to the mathematical needs of those using it.

There is sufficient evidence to show that students are receiving credit for mathematics which they have not mastered sufficiently for use in situations demanding it.

And this brings up the third possibility: Are the mathematical situations in physics so different from those learned in mathematics that the students fail to recognize them without assistance? The elaborate studies by Carter,[7] on the secondary school level, and Congdon,[8] on the college level, have shown how closely mathematical concepts, processes and methods of procedure are interwoven with the physics in textbooks. Of particular value here, also, are the results of an extensive analysis of the mathematics in laboratory manuals prepared especially for local use in the first courses in college physics. For this study,[9] 72 local manuals, containing 3117 experiments, were gathered from 61 colleges and universities. Since these "home made" manuals, were prepared with local situations and specific objectives in mind, they probably conformed more nearly to the type of mathematics actually used in the physics course than that in the textbook, which is seldom completely studied in any one course. The results of this investigation are here condensed for convenience.

Laboratory physics was found to be even more quantitative than textbooks. The range of mathematics used extended from very elementary algebra, in the least difficult manuals, to very simple calculus, in the most difficult manuals. The usual year and one-half of algebra and one year of plane geometry in secondary school covered practically all the items found in the manuals for these two subjects. The use of solid geometry was almost nil. Little, in trigonometry, beyond the use of the sine, cosine and tangent, was found. Analytics was used in less than 15 per cent of the manuals, but even in the most difficult engineering manuals, its use included just the bare elements of the rectangular hyperbola. In most cases, however, the analytics of the straight line was as far as these manuals went. Graphical representation was extensively used. In fact, this was so important that more than one-half of the manuals devoted from one to six pages to a discussion of it in their introduction.

[6] Lueck, W. R.: *The Arithmetical and Algebraic Disabilities of Students Pursuing First Year College Physics*. Contributions to Education, VIII, No. 1, University of Iowa, 1932.

[7] Carter, W. R.: *Mathematics Teacher*, XXV: 313, Oct., Nov., Dec., 1932.

[8] Congdon, A. R.: *Training in High School Mathematics Essential for Success in Certain College Subjects*. Contributions to Education, No. 403, Teachers College, Columbia University, 1930.

[9] Oerlein, K. F.: *Op. cit.*

An analysis of the mathematical words and expressions used in these manuals served to further emphasize the quantitative nature of laboratory physics as compared with the textbooks. The manuals furnished abundant examples of the *functionalized* concept of mathematics. Summing up, two words characterized the mathematics in the manuals; it was analytical and functional.

Dodge,[10] in compiling the opinions of a number of directors of industrial research laboratories, states:

The opinion is that physicists usually take sufficient work in mathematics. The complaint is made that the courses, in many instances, are not satisfactorily taught. . . . Clearly, we have here a problem that will have to be worked out in cooperation with departments of mathematics. . . . It is unfortunate that many departments of mathematics are so little concerned with the applications of their science to physics, for the satisfaction of whose demands most mathematics has been developed. It is somewhat as if we had departments of the Dictionary staffed by men largely ignorant of literature and convinced that their beloved words would be contaminated if strung together for such ignoble and practical purposes as the making of sentences endowed with meaning . . . the need is for better mathematics rather than for more.

Evidence is thus accumulating that there is considerable truth in the third possibility. The studies seem to show that the actual number of different mathematical items used in these first courses in physics is much smaller than had been, at first, anticipated. And this in a course conceded to be, with the exception of mathematics itself, the most mathematical course in the freshman year! But, although these relatively fewer items occur many, many times over, changed physical settings require changed mathematical forms. Fewer items need to be mastered in more varied situations.

Here, then, lies the crux of the problem. Too much, rather than too little mathematics has been taught. *To teach for real mastery, fewer items, carefully selected on the basis of use, without regard for the formal divisions of mathematics, might well be the aim of a thorough revision of secondary school mathematics.*

To some it may come as a distinct surprise to learn that such revised courses have already been worked out. The Ninth Yearbook,[11] contains such a course based upon the functional concept of mathematics, which, if put into practice would delight the heart of any physics teacher. Most discouraging, however, is the general attitude of mathematics teachers toward these revisions of their field. The opinion is that mathematics teachers have not been as responsive to the mathematical needs of others as they ought to be. Unless these "functionalized courses," or their equivalent, seep into our secondary schools faster than they have so far, mathematics teachers are likely to continue under the censure of being uncooperative and self-satisfied.

That mathematics must, of necessity, befriend and cooperate with physics was once again illustrated by a very recent situation. A proposed curriculum for the first two years of college was presented for discussion. It appeared well balanced, with but one glaring omission. No mathematics was indicated! With the exception of the mathematics group, the only one to voice a protest was the physics teacher.

There is no intention here of releasing the physics teacher of some responsibility for student failure arising from the mathematical nature of the course. Physics teachers must continue to assist students in making the necessary transfer of their general mathematical knowledge to the specific physical situations. Just where this general information should end and transfer begin is a matter calling for a most sympathetic and close cooperation between the two departments. How much longer can an artificial departmentalization forestall a natural cooperative movement between these two related fields?

[10] Dodge, Homer L.: *Am. Physics Teach.*, IV: 169, Dec., 1936.

[11] Hamley, H. R.: *Ninth Yearbook*, The National Council of Teachers of Mathematics, Bur. of Publ., Teachers College, Columbia University, 1934.

THE ART OF TEACHING

A NEW DEPARTMENT

A Unique Mathematics Exhibit

By RUTH WILSON
Thomas Jefferson High School, Richmond, Virginia

REALIZING that many people, even school administrators, regard mathematics beyond arithmetic as a subject with neither cultural nor practical value, we of the mathematics department of Thomas Jefferson High School decided to make the topic of our second annual exhibit: "The Practical Application of Mathematics in Various Occupations and Industries." We knew that mere statements of facts would receive little attention—there must be something to attract the eye, and we felt that the statements would be more convincing if they came from business men. Our first problem, therefore, was to devise a pleasing scheme for getting the attention of the public in order to sell the idea that all high school mathematics is practical and to stimulate appreciation of the fact that nearly all "big business" uses higher mathematics. Our next problem was to secure the cooperation of various business concerns.

The whole department went to work. We made a partial list of firms which use mathematics in some of their departments. Then we tried to get in touch with each firm. If it was a Richmond concern, one of the teachers called personally on an official of the company, explained the purpose of our exhibit, and asked what he could contribute in the form of a chart. This, at first, was a difficult task, as we had never seen the kind of chart that we were asking for, and, not being business experts ourselves, it was impossible to describe just what we wanted. We persisted, however, and finally made a creditable start.

I called on the superintendent of one of the divisions of the Virginia Electric and Power Company, and explained, as best I could the sort of thing I wanted. This man happened to be an old pupil of mine and was an enthusiast on the subject of mathematics. It chanced that at that very moment he needed five new men in his division, and, while there were plenty of applicants, both high school and college graduates, there were not five with a working knowledge of higher mathematics. He became keenly interested in our idea concerning the exhibit and promised to have something made for it. Immediately he went to work to carry out his promise. He interested other officials in the company, and they spent a day in selecting from their files pictures representing each division of the service. Then they wrote out a typical problem used by each division with the formula for its solution. Their artist was put to work on the chart with instructions to do his best, and the result was a beautiful poster, four and one-half by three feet, on a blue background with lettering in black, white, and red. Under each picture was placed a typed card containing the selected problem and formula. One of these was from calculus; three were algebraic. The problem used by the division of transmission of power involved trigonometry and the one from the sales division arithmetic.

With this chart to show to other firms, it was not so difficult to explain what we wanted. They all received us with the most courteous attention, and showed great interest in the project. Two of the

corporations which made us most attractive charts were the Chesapeake and Potomac Telephone Company, and the Atlantic Life Insurance Company. Both of these contained statements about the importance of mathematics, and certain formulas used by the company. The insurance company's chart was mounted on a large easel. The General Motors Company, at our request, made us a three by two foot reproduction of a page from one of their pamphlets, showing a photostatic copy of a leaf from the log book of the engineer who worked on "knee action," with the various mathematical calculations used. The city engineering department of Richmond made us a series of charts called "Mathematics in Map-Making." These were most attractive and explained how surveyors use geometry and trigonometry in triangulation. They also gave us four maps of Fort Harrison, showing stages in development from the skeleton outline to the finished map.

A local architect made a most interesting contribution to the exhibit. He gave us a drawing of the front steps of our own school, with an explanation of the mathematics involved in the construction. He also gave us a perspective in colors of one of the buildings of the College of William and Mary and an attached diagram showing how trigonometry was used. The same architect contributed a blue print of the new William and Mary amphitheater with a mathematical explanation.

An instructor at the United States Naval Academy donated a chart headed "A Practical Problem in Navigation," stating the problem, and giving two solutions—one trigonometric, the other graphical. A United States Army officer made us a similar chart, explaining the "three-point problem" and telling its advantages in locating gun positions. The Corning Glass Works and the Bailey Meter Company of Chicago sent us materials and directions for making charts, and we made these ourselves. The title of the one from the Glass Works was "Some Mathematical Problems Encountered in Making a 200-inch Telescope Disk."

Realizing at the last minute that we had no graphs in our exhibit, we called at the Richmond offices of the R. F. & P. Railroad, and asked whether they had any that we might use. They gave us three large graphs, showing their maintenance costs, their expenditures and revenues, and the trains operating between Richmond and Washington on a given day. They said that if we had another exhibit, their engineering department would be glad to work up a chart showing how they put all mathematics to practical use.

Other articles which we showed were a large framed picture of the "Tree of Knowledge," advertised in the *Mathematics Teacher* for May 1936; a chart made in the mechanical drawing class with the quotation from Betz's book, beginning, "Mathematics is the Key, and Applied Mathematics is the Tool wherewith Man Conquers the Universe"; a card with the question, "Is Mathematics Necessary?"; and a collection of "Career" pamphlets about occupations which require a knowledge of mathematics. We also displayed a diagram of our school building and grounds, a project planned and executed by the transit committee of our mathematics club. Still another feature was a collection of letters from business men and deans of colleges, telling what they thought about the importance of high school mathematics, especially trigonometry and solid geometry.

The exhibit was displayed in our school library, where it was inspected by the high school students and faculty, members of the school board, the superintendent and assistant superintendent, and mathematics teachers from other high schools. At the request of our school librarian, we left it up during the week of the Librarians' Convention in Richmond. The following week it was shown in two large windows of a downtown publishing firm, where it attracted much attention and favorable comment.

The Relationship Between Silent Reading Ability and First Year Algebra Ability

By Guy E. Buckingham
Allegheny College, Meadville, Pennsylvania

It has been shown by the author[1] and his colleagues, Hottenstein, Gilliland and Tucker that a positive relationship exists between one's success in first year Algebra and his ability to eliminate errors in the simple fundamental processes of addition, subtraction, multiplication and division. It has also been shown by the same group[2] that these errors persist in the more complex processes of Algebra attempted during the first year.

From an inductive approach it follows that many factors involved in the learning of Algebra should be investigated. The more factors interfering with the process which can be eliminated the more efficient the process should be. Among these factors silent reading comprehension is prominent. Little is known concerning the relationship between the ability to comprehend paragraphs with a non-technical vocabulary and the ability to learn Algebra.

In May, 1933, the Gates Silent Reading Tests for paragraph comprehension, types A, B, C, and D were administered to 105 freshmen in the Meadville, Pennsylvania, High School. All of these students were finishing their first year of Algebra. The Coöperative Algebra Test, Form 1933 was given to the same group.

These students may be described as the usual freshmen found in a third class city high school, following the 8–4 plan of administration. They were taught by three experienced and properly certified teachers all with four years of training beyond the secondary school. Each teacher has had professional training and maintains a good professional attitude. The scores on the Otis Self Administering Test of Mental Ability indicated a normal group mentally.

Test Results

The Gates Silent Reading Test, Type A, is designed to measure one's ability to appreciate the general significance of a paragraph. The mean score of the group on this test was 17.0 with a standard deviation of 3.91 and a range of 7 to 24.

The Gates Silent Reading Test, Type B, is designed to measure one's ability to predict the outcome of given events. The mean score on this test was 19.5 with a standard deviation of 3.46 and a range of 9 to 24.

The Gates Silent Reading Test, Type C, is designed to measure one's ability to understand precise directions. The mean score of the group on this test was 18.8 with a standard deviation of 3.66 and a range from 7 to 23.

The Gates Silent Reading Test, Type D, is designed to measure one's ability to note details. The mean score on this test was 47.8 with a standard deviation of 7.1 and a range from 27 to 54.

The Coöperative Algebra Test, Form 1933, measures one's ability in elementary Algebra through quadratics. The mean score of the group on this test was 37.5 with a standard deviation of 11.2 and a range from 15 to 70.

Interesting Scores

The student with the highest score in Algebra had a reading grade of 4.8 in ability to note details.

The student with the lowest score in Algebra had a reading grade of 8.9, essentially normal reading ability, with respect to noting details.

[1] Diagnostic and Remedial Teaching in First Year Algebra. Northwestern University, School of Education, Monograph No. 11, Evanston, Illinois, pages 83–95.

[2] Ibid. Pages 68–82.

The student with the highest score in Algebra had an I.Q. of 120 and the student with the lowest score had an I.Q. of 98.

Many students had reading grades of 3, 4, and 5 in ability to appreciate the general significance of a paragraph. Thirty-nine per cent of the whole group had less than seventh grade ability in this test, and seventy-nine per cent had less than ninth grade ability. The mean grade score of the group with less than ninth grade ability was 6.48.

Over fifty-five per cent of the total group were under grade with respect to ability to predict the outcome of given events. Some scores went as low as grade four. The mean grade score of those under grade was 7.38.

More than forty-six per cent of the total group were under grade with respect to ability to understand precise directions. Scores indicating reading grades of five and six were quite frequent. The mean grade score of those below grade was 7.12.

Over fifty-two per cent had scores below grade in reading to note details. The mean grade score of this group was 7.04

A student with an I.Q. of 108, accelerated in all four types of reading from one to two years, had an Algebra score of 17. The lowest score of the entire group was 15.

A student with an I.Q. of 76, retarded from one to five years in the various tests, had an Algebra score of 19.

Group Correlations

The correlation between Type A reading and the Algebra scores was $.30 \pm .059$; between Type B reading and Algebra scores $.41 \pm .053$; between Type C reading and Algebra scores $.38 \pm .056$; and between Type D reading and Algebra scores $.11 \pm .065$.

Part II of the Coöperative Algebra Test is composed of statement problems. The correlation between Type A reading and the scores on part II was $.26 \pm .064$; between Type B reading and part II $.21 \pm .066$; between Type C reading and part II $.39 \pm .058$; and between Type D reading and part II $.22 \pm .065$.

Findings

1. The mean score of the group in reading to appreciate the significance of a paragraph was 17.0 which indicates a reading grade of 8.0 or two years of retardation.

2. The mean score of the group in reading to predict the outcome of given events was 19.5 which indicates a reading ability slightly below grade.

3. The mean score of the group in understanding precise directions was 18.8 which indicates a reading ability slightly below grade.

4. The mean score of the group in reading to note details was 47.8 which indicates ability slightly above grade.

5. The mean score on the Coöperative Algebra Test was 37.5 which is some higher than the mean score based on the scores of 2,144 pupils in 53 public high schools. The sigma of 11.2 is smaller than the sigma of the scores of the 2,144 pupils which indicates that a considerable proportion of the group of this study were above the mean score of the 2,144 pupils.

6. Large percentages of the total group were below grade in all types of reading ability.

7. All correlations between the total Algebra scores and the different types of reading ability were positive and relatively low. There was more correlation between ability to predict outcomes and follow directions and Algebra scores than between ability to appreciate the general significance of a paragraph and note details, and Algebra scores.

8. Logically the most startling result was the amost negligible correlation between ability to note details and Algebra scores.

9. All correlations between the different types of reading ability and the ability to solve statement problems were positive and relatively low.

10. It is rather startling that the correlation between ability to follow directions and ability to solve statement problems is so low. However, it is the highest correlation among all the types of reading and the results in Algebra.

Interpretations and Questions

1. Ability to read with a non-technical vocabulary is a necessary quality for achievement in Algebra.
2. The correlations are so low that there must be other necessary abilities present in order to achieve well in Algebra.
3. Could these other qualities be concerned with vocabulary?
4. Could they be concerned with a reading frame of reference peculiar to Algebra?
5. Could they be concerned with the student's application or the manner of teaching?
6. If a student with good intelligence can lead his class in Algebra and simultaneously possess fifth grade ability in reading to note details, would a higher ability in this respect cause him to improve his Algebra score?
7. Is the result worth the effort and cost to either society or the student, with an I.Q. of 76 and two to five years retardation in silent reading, who has the lowest score in his class? Is there no such thing as a law of diminishing returns in Algebra?
8. Why should a student with normal intelligence and considerable acceleration in reading ability be at the bottom of his class in Algebra?
9. Is it possible that we are confronted not with generating ability to read, but rather ability to read Algebra?
10. May this condition be possible for all secondary school subjects?

The Educator's Fair

Tune: "I went to the Animal Fair"

I went to the Educator's fair
The Frontier Thinkers were there
They handed out dope
Of boundless scope
All based on a questionnaire.

These thinkers who thought they thunk
Said all subject matter was junk
Said the schools of the nation
Must have integration
But it sounded to me like the bunk.—

A Victim

EDITORIALS

Contributions of Mathematics to Civilization

The National Council of Teachers of Mathematics is inaugurating a plan of publishing a series of monographs to be known as "Contributions of Mathematics to Civilization." These monographs are to be given to each person who is a member of the National Council at the time the monograph appears. They will be sold postpaid to others who are interested in having them at twenty-five cents each.

The first of the series, "Numbers and Numerals—A Story Book for Young and Old," was written by Professors David Eugene Smith and Jekuthial Ginsburg, both scholars in the history of mathematics, and is now ready for mailing. The second monograph on "Great Men of Mathematics" will appear in due time. It is planned to devote the third monograph to "The Story of Measurement." These monographs should be useful not only to teachers of mathematics, but also to teachers of the social studies who may wish to use them in their classes as supplementary reading material. Why, for example, should history teachers spend time on the "war lords" when the contributions of men like Descartes, Newton, and Pasteur to civilization are so much superior. It is hoped that this series will meet a real need in the classrooms of this country.

Moreover, the plan is to get the man on the street who has a mathematical bent interested in these contributions by making these monographs available at a price within the range of his pocketbook. The trouble with such books and monographs, where they exist at the present time, is that the cost is exorbitant for the ordinary person. If the majority of our people and even many teachers of mathematics are ever to read about the contributions of mathematics to civilization we must provide books at popular prices. However, these monographs are issued at considerable expense and if the plan is to succeed, members of the National Council who receive a free copy should use it to inform others of its value so that each edition may be completely sold out. A complete announcement of the first monograph appears on the back cover of this issue.

What Is a Mere Classroom Teacher to Think?

The senior high school course is now the least satisfactory part of the entire six years of the secondary school. We are pretty well agreed on the work of the junior high school in so far as content is concerned except that in some schools and in some textbooks we still retain too much obsolete material or material that is too difficult. It is fairly well agreed among the leaders both in mathematics and in secondary education that the course in the junior high school should be of a general mathematics nature and that it should be required of everybody. In any case then, whatever mathematics is taught in the senior high school can be made elective. However, it seems clear that before we can decide what the mathematics course of the senior high school should be, we must first determine of what the course in general education should consist and then decide what the place of mathematics and the other great fields of knowledge should occupy in that scheme. One difficulty confronting classroom teachers in secondary education is that educational leaders do not agree. On the one hand a man of the standing of Chancellor Chase of New York University says[1]

[1] Chase, Harry Woodburn, *American Mercury*, November, 1934.

Economics, modern history, sociology, philosophy, basic ideas about the natural sciences, some knowledge of the fine arts, are today a far more necessary part of the equipment of the cultured laymen than is the merely formal study of languages and mathematics. . . . The idea that liberal education should be a unity, based on some coherent philosophy of life, has become obscured. Once it was a unity, centered about the classics. It gave to a few a type of culture that marked them off as a class apart. Now it must unify itself again, not about the classical cultures, but about the problems of contemporary life.

On the other hand, Dr. Hutchins, the President of one of our greatest Universities, says[2]

The tradition in which we live and which we must strive to help our students understand and clarify is hidden from our sight because of our own defective education. We are all the products of a system which knows not the classics and the liberal arts. There is every indication that the system is growing worse instead of better.

Every day brings us news of some educational invention designed to deprive the student of the last vestiges of his tools and to send him for his education helpless against the environment itself. The worst aspects of vocational education, progressive education, informational education, and character education arise from the abandonment of our tradition and the books and disciplines through which we know it.

. . . .

The arts central in education are grammar, rhetoric, logic and mathematics. The liberal arts are understood through books and books are understood through the liberal arts. The tradition is incorporated in great books. The teachers of English are the last defenders and exponents of these books and of the arts of language.

If we are to really solve some of the most pressing problems of American education, the leaders in education should try to find some kind of common ground upon which they can agree not only with reference to the content, but also with respect to the pupil and the teacher. Many of our troubles would be over in this country if we had a well trained and scholarly group of high minded teachers, but until they are all more or less like that the leaders in education should try to coöperate a little better than they do in helping teachers to know, all things considered, what the best practices are.

[2] *N. Y. Times*, January 31, 1937.

Joint Report on the Teaching of Mathematics

THE General Education Board has awarded a grant of five thousand dollars to the Joint Commission of the Mathematics Association of America and the National Council of Teachers of Mathematics on "The Place of Mathematics in Secondary Education." This grant will enable the Commission to prepare a first-class report. The Chairman of the Commission is Professor K. P. Williams of Indiana University. A two-day session of the Commission was held at the Palmer House after the annual meeting of the National Council of Teachers of Mathematics was adjourned on February 20th. All members of the Council should do everything to support the work of this commission.

W. D. R.

◆ IN OTHER PERIODICALS ◆

By Nathan Lazar
Alexander Hamilton High School, Brooklyn, New York

Arithmetic and Algebra

1. Curfman, L. E. *The slide rule in the solution of cubic equations.* Bulletin of the Kansas Association of Teachers of Mathematics. Vol. 11, no. 1. October 1936, pp. 4–5.

A description of a method of solving the cubic equation by means of a specially constructed slide rule. The paper is based on an article by Russel A. Whiteman that appeared in "Civil Engineering," October 1934.

2. Fletcher-Jones, A. A. *A method of long division for small divisors.* The Mathematical Gazette. 20: 331–32. December 1936.

An interesting method of performing long division operations that depends on the binomial expansion of a negative power.

3. Nogrady, H. A. *A new method for the solution of cubic equations.* American Mathematical Monthly. 44: 36–38. January 1937.

The author describes a method of using a table that he invented to facilitate the solution of an equation of the third degree.

4. Pease, Daniel. *A present day algebra for tomorrow.* Bulletin of the Kansas Association of Teachers of Mathematics. Vol. 11, no. 1. October 1936, pp. 6–8.

"This paper is intended to set forth the need for teaching algebra in such a way that algebra not only benefits the student today, but tomorrow as well."

5. Shain, Julius. *The method of cascades.* The American Mathematical Monthly. 44: 24–29. January 1937.

An exposition of method, invented by Rolle for the general solution of numerical equations of the form

$$x^n+a_1x^{n-1}+a_2x^{n-2}+\cdots a_n=0$$

The author illustrates the method by working out a typical example and points out its historical significance.

6. Smith, E. V. *The teaching of indices and logarithms.* The Mathematical Gazette. 20: 324–26. December 1936.

Interesting comments on the possibility of teaching logarithms by having the pupils make a paper slide rule for multiplication and division.

7. Stelson, H. E. *A comparison of methods for finding the interest rate in installment payment plans.* National Mathematics Magazine. 11: 172–76. January 1937.

After analyzing the various methods used in current text books for finding the rate of interest in installment payment plans, the author concludes that many approximation formulas "which have been either used with the idea that the simple interest itself or an approximation is obtained are not very close or consistent approximations. Their use can be justified only where very crude results will be satisfactory, and where more intricate formulas are too difficult for the user."

Geometry

1. Bubb, Frank W. *On the role of geometry in science.* School Science and Mathematics. 37: 55–71. January 1937.

By means of many detailed examples culled from the history of physics and of mathematics, the author argues that ". . . Geometry, the first and noblest of sciences, has played in the past and will probably continue to play the dominant role in science. It certainly dictates the form and to some extent the content of all scientific concepts. It is the stage upon which those abstract actors, the causes and effects, play their scientific roles. It is the picture frame wherein that greatest of all artists, the creative scientist, paints his pictures."

The thesis of the article is marred, however, by philosophic views which are not only naïve but outmoded as well. Thus the writer believes that "most of what he [the scientist] knows has not come to him through his sensory channels; he has invented most of it. His senses are like a pinhole camera. Through this pinhole he peers out at the world. And just how tiny this pinhole is, is rather frightening—makes one feel like a child lost in the dark when one realizes the vast extent of the reality which one seems barred from ever perceiving directly."

That the above fear is not a genuine one can be demonstrated by the following counter-question: Since the senses are the only way of getting acquainted with "reality," how does the writer know that there is any other reality from which he is barred?

2. Stone, Charles A. *The place of plane geometry in the secondary school curriculum.* School Science and Mathematics. 37: 72–76. January 1937.

The report of a study undertaken to determine the advisability of postponing the year in which the study of geometry is begun. After outlining the procedure followed and the tests administered, the author concludes that "Certainly the results present definite evidence that plane geometry should have a place somewhere beyond the second year. Perhaps the senior year is the place for it. The writer believes that in the senior year the solid geometry could be completed, in addition to the plane geometry, with greater understanding and facility. Likewise, the algebra could be postponed, and the equivalent of one and one-half years of algebra could be completed in the junior year. Or perhaps experimentation would result in reversing the above, putting the geometry in the junior year and the algebra in the senior year. Those in charge of mathematics should take cognizance of these facts, or else we may find algebra and geometry being moved up into the college, where they were previously taught. They should remember that algebra and geometry were originally intended as studies for adults; they were organized to meet the needs and abilities of all the people and in no sense of the pupils who are studying them today."

Miscellaneous

1. Emch, Arnold. *Rejected papers of three famous mathematicians.* National Mathematics Magazine. 11: 186–89. January 1937.

The author discusses "Three outstanding examples of extremely meritorious papers which were rejected by supposedly competent critics in editorial or academic committees. This does not mean that these are the only known cases, because there is no doubt that many other instances could be mentioned in support of these curious historic events."

The following are the three rejected papers discussed:

1. Schläfli: Theorie der vielfachen Continuität.

2. Reimann: Commentatio mathematica qua respondere tentatur quaestioni ab Ill^{ma} Academia Parisiensi propositae.

3. De Jonquières: De la transformation géométrique des figures planes.

2. Kempner, A. J. *Anormal systems of numeration.* The American Mathematical Monthly. 43: 610–17. December 1936.

A welcome addition to the literature dealing with the possibility of using other numbers than ten as a base for building a number system. In addition to many interesting theorems and proofs, the following note is supplied: "The numbers 0 and 1 are clearly excluded as bases. Positive numbers less than 1 and negative numbers may be used as bases with slight modifications of the process and suitable restrictions on the set of digits employed."

3. King, Ronold. *The elementary foundation of mathematical physics.* The American Mathematical Monthly. 44: 14–22. January 1937.

The author analyses the elementary foundation of mathematical physics in three major parts. "These are, first, the experimental technique which makes it possible to inter-compare natural phenomenon to a high degree of precision; second, the framework of permanent relations which is discovered in experimental fact and which is expressible in terms of mathematical functions; third, the vast and highly articulate system of mathematical logic. Upon this foundation the human mind has constructed its most powerful and most dependable tool—the mathematical-scientific method. It is used wherever mathematics serves a practical purpose. It is the leaven of modern knowledge."

4. Nicklin, Esther. *A short history of linear measurement.* Bulletin of the Kansas Association of Teachers of Mathematics, Vol. 11, no. 1. October 1936, pp. 8–11.

A summary of the various units of linear measurement used throughout the history of mankind. A short bibliography of seven items is included.

5. Schorling, Raleigh. *A fundamental problem.* Bulletin of the Kansas Association of Teachers of Mathematics. Vol. 11, no. 1. October 1936, pp. 2–3.

This article deals with problem of the slow-learning pupil, and sets forth the "reasons why the schools should set about immediately to build appropriate curriculums for the lower levels of ability in the high schools."

6. Richeson, A. W. *Notes on a seventeenth century English mathematical manuscript.* National Mathematics Magazine. 11: 165–71. January 1937.

After commenting on the authorship and the probable date of the manuscript, the writer reproduces a few problems and diagrams found therein. "The contents and subject matter of the manuscript would seem to indicate that the manuscript followed a definite course of instruc-

tion given at some college or university. Frequently the topics overlap each other and there is a certain amount of repetition in the type of problems, examples, and theorems. In many places there is no attempt to prove the theorems nor even to state them, but merely to give one or two numerical examples involving some unstated rule, as in the discussion of the trigonometric solution of the triangle. The definitions are, for the most part, given in simple language, and are followed by numerous examples, figures and diagrams illustrating the definitions and text matter. On the whole, however, the manuscript is well written."

7. Wood, Frederick. *Sectioning students on the basis of ability*. National Mathematics Magazine. 11: 191–94. January 1937.

A report of an experiment in grouping of students according to ability, conducted in the mathematics department of the University of Nevada. After a detailed exposition of the plan followed, the author concludes that from all the evidence that he "can collect here, by the opinions of the faculty and students, by the progress of students in subsequent courses, the conclusion is that for the University of Nevada sectioning according to ability in mathematics is much to be preferred to any other method."

NEW BOOKS

General Mathematics. By W. S. Schlauch, F. S. Crofts and Company, 1936, IX+393.

This new book is a text in *general mathematics* containing the kind of mathematical training that will furnish the student with the type of background material that is prerequisite for successful work in "the mathematical theory of investments, finance, insurance, business calculations, statistics, and budgeting." The illustrative problems and exercises used in the book are taken mostly from the business world so that the work at once has a practical setting.

The high standing of the author both as a scholar in his field and as a very successful teacher guarantees a careful selection of material suited to the need for such a course in schools of business.

The author has outlined the work in three alternative courses; one for a course of two hours a week for thirty weeks, and the second for three hours a week for thirty weeks, and the third for four hours a week for thirty weeks as the circumstances in a given school permit.

The book is well written and should receive the attention of all teachers in the field of business.

W.D.R.

Mathematics of Everyday Life. By G. A. Boyce and Willard W. Beatty, Inor Publishing Company, 1936.

Health Unit, XIII+130, Price $0.72
Finance Unit, XII+88, Price $0.62
Geometry Unit, XIII+168, Price $0.87
Leisure Unit, XIII+156, Price $0.73
Drill Unit, XIV+181, Price $0.82

As the name *Mathematics of Everyday Life* suggests these five units of work deal with measurement of the social and economic environment of pupils. It is the feeling of the authors that the first four units offer abundance of work for a full two-year course in junior high school mathematics. The drill unit is intended for supplementary drill work with each of the other units.

The authors claim that these units "broaden the scope of mathematics from the narrow field of mechanical computation to include the more interesting and useful field of quantitative thinking." If they can really do this, they will serve a useful purpose in the education of children.

Some time ago, we had books of a similar type which were called "Fifteen Weeks" of this or the other topic, but they were only a passing fad. The question now becomes, is there a new demand and a need for books of that type rather than a careful and sequential development of teaching material. Since the proof of the pudding is in the eating, it will be interesting to see how these units will be received both by teachers and pupils. The idea of getting rid of obsolete material and connecting mathematics up with real life problems is sound and should be encouraged. The method of doing it may be open to debate. Teachers may wish to know just where and how these units are to be used. No doubt the authors can supply this information.

W.D.R.

Mathematics Through Experience. By J. S. Georges, R. F. Anderson and R. L. Morton, Silver Burdett Company, 1937.

Book One, VI+378, Price, $1.00
Book Two, VI+394, Price, $1.04
Book Three, to be ready April 1, 1937.

These new books are organized on the unit plan, but instead of centering around Health, Finance, Leisure and other topics they are organized in comprehensive units of learning along psychological lines. The instructional materials in each unit lead to an understanding of the elements of the unit and in turn the elements are related to the central theme of the unit. Tests are interspersed throughout each book for evaluation purposes and also to furnish the teacher with an idea of how similar tests may be constructed.

The books represent the kind of general mathematics approach which is coming to be the most desirable solution to the problem of what mathematics to teach in the junior high school. The first book starts out in an interesting way by familiarizing the pupil with common geometric concepts and after four chapters takes up arithmetic for awhile and then in the last four chapters returns to more geometric work. The second book starts out with algebra and after four chapters shows the pupil how algebra and geometry "cooperate."

The books are written by men of rich experience, are attractive in appearance, and should be carefully considered by all teachers who are looking for the modern type of mathematics course for the junior high school.

W.D.R.

Mathematics in Life. By Raleigh Schorling and John R. Clark, World Book Company, 1935.

Unit A. Measurement, IV+44, Price, \$0.24

Unit B. Constructions, IV+60, Price, \$0.28

Unit C. Drawing to Scale, IV+44, Price, \$0.24

Unit D. Per Cents, IV+60, Price, \$0.28

These pamphlets are intended to give the non-mathematically minded pupil a type of treatment of content that will be of interest and use to him and which is not beyond his ability to comprehend. Activities and practice are woven together as the topics are developed so as to form a kind of laboratory procedure.

Emphasis is placed upon the learning side, the material is said to be graded in small steps and so arranged as to facilitate reading. These pamphlets along with other similar attempts to interest and help the slow moving pupils should be studied by those teachers who are desperately trying to find some kind of useful content in mathematics which will hold the attention of hundreds of pupils for whom the more abstract elements of formal algebra are a nightmare. Can such units solve the problem? Even if they can help materially they will be worth while. There should be some way of pooling the experiences and testimony of teachers who have a chance to use such material.

W.D.R.

Geometry of Space and Time. By Alfred A. Robb, Cambridge at The University Press: New York: The Macmillan Company, 1936, 408, Price, \$7.50.

Unfortunately the subject of geometry is such that the first few steps, and really the most important ones, are the most difficult. It is an almost superhuman task to hurdle the difficulties that are present when a student of geometry has to master the first few fundamental ideas upon which the structure of geometry is built. The point, another point, their relation to a third point, etc. Trite, one would say perhaps. But let one try to develop a logical structure without the rigorous treatment of these ideas, and he is in trouble.

There are a few treatises in which the foundations of Euclidean Geometry are fully presented in the sense that would satisfy a most exacting logician in this year of 1937. But do secondary school teachers in this country ever trouble themselves to consult Hilbert, Veblen, or Forder? If one is to bewail the fact that geometry is on the way out, let him pin the blame on the textbook writers and on poor teaching. The present trouble with our geometry lies in the fact that neither the textbook writers nor the teachers (by and large) have sufficient vision to go beyond the traditional Euclid.

How many secondary teachers ever thought of the possibilities and opportunities that the coordinate method, combined with rudimentary knowledge of mechanics on the high school level might offer for the enrichment of the pupils' work in mathematics? Let no one be frightened by the fearsome title "Geometry of Time and Space." It is less formidable than it sounds. A resourceful teacher would find considerable (and profitable) pleasure in developing the geometry of a particle moving along a straight line (that is, the geometry of a line-time world). From one point of view more meaning would be attached to the science taught in the schools, from another point of view the pupils would see that Euclid's geometry is not the only one that requires strict logical development. The scope of mathematics necessary for such a development can be limited to intermediate algebra.

Dr. Robb's book offers an opportunity for those who might be interested in enlarging and enriching the views to see what one could do with *geometry* if sufficient vision and imagination were present. This is a very interesting and instructive book. It uses notions that are common to all. *Before, now, after, simultaneity;* these are ideas that would excite any pupil and would, no doubt, lead to interesting classroom situations, if properly presented.

This treatise of Dr. Robb's presents a logical development of these notions considered from a geometric point of view. Whether we use the classical Newton-Gallilean system or the Minkowskian time-space, all depends on our point of view. However, the use of the Minkowskian system invalidates certain properties of Euclidean geometry. Dr. Robb makes this clear in a very comprehensive manner.

It is difficult to point out the most outstanding features of this treatise. It is a monumental work, a mine of material, that *should not be merely read*. It requires careful study. It is not difficult to master, if one desires to delve into the subject. It should be made a part of the reference in every course in professionalized subject matter in geometry for teachers.

A. Bakst

Bibliography of Early American Textbooks on Algebra. By Lao Genevra Simons. The Scripta Mathematica Studies, No. 1. Price, \$1.00

This bibliography of algebra textbooks printed in the Colonies of the United States by the middle of the nineteenth century grew out of a study of the introduction of algebra into the American colleges in the eighteenth century.

The author assembled her material, with a few exceptions, from the books on the shelves during actual visits to the libraries cited in connection with each work.

The variety in pedagogical approach as shown by the study "is enough to suit any tastes; arithmetic before algebra, algebra before arithmetic, and even algebra before geometry, a radical change, indeed, at that time."

This study gives evidence to support the belief that there is "nothing new under the sun." For example, the author brings out the point that a hundred years ago in 1837, Benjamin Peirce included fundamental notions of calculus in his algebra. Similarly, with respect to other innovators.

This Bibliography should have a place in the library of every teacher interested in the history and teaching of mathematics and should especcially be in all teacher training libraries.

W.D.R.

Practical Algebra—Introductory Course. By Clifford Brewster Upton. American Book Company, 1937. vii+488 pp. Price, $1 28

When the long-lived arithmetics of former days were brought out in new editions, they frequently carried the subtitle "Very much enlarged and expanded." This would surely apply to the *Practical Algebra* and the *Modern Algebra* (1930) on which it is based for the new volume is more than fifty per cent larger than its predecessor. The earlier book had important characteristics which have been preserved in its successor: the careful step-by-step development of its topics, and the mechanical detail of its unit page and its effective use of italics.

The new book does not look like a text book and one suspects that pupils may prefer its striking binding to the more conservative cover of its predecessor. The end papers suggest that the material has been stream-lined throughout for these are a conglomerate of modern power and transportation.

There have been certain important changes in order. The pages on graphs have been removed from the first chapter (Formulas) giving it greater unity. The introductory chapter on equations remains substantially as before. A treatment of bar and circle graphs with a more extensive discussion of line graphs makes up a new third chapter. The computational detour of special products, fractions and fractional equations now comes in what would normally be the second half year's work thus allowing a more extended treatment of formulas and the chapter on sets of linear equations to follow closely on the chapter on linear equations. The chapter on sets of linear equations has been given an introductory section on solving motion problems graphically which is an important addition.

One criticism of the earlier book was lack of practice materials. This comment, if it was valid has been amply provided for as will appear by noting the exercises that have been added. Curiosity led me to make a count in the chapter on equations. The additions totalled over 300. A system of asterisks has been adopted to indicate the more difficult exercises and certain sections are plainly designated as being optional. The review at the close of the book has been expanded from six to twenty two pages and is more comprehensive. Diagnostic tests have been added to each chapter and these are keyed with page references to exercises that might be repeated for remedial work. Occasional pages of lists of important words and expressions are a great assistance to the student. It should not be inferred, however, that the greater number of exercises have made the volume dull and heavy. On the other hand, the rearrangement mentioned above probably results in great saving of time and effort by postponing the parts of the work that involve more technical computation and by associating more closely the skills gained in the solution of simple equation.

In many ways, the *Practical Algebra* is more direct and appealing than the *Modern Algebra*. Instead of beginning with a discussion of "The Age of Electricity" on the first page and mentioning the formula $W = VA$ about which the student probably knows nothing, the new volume starts with "Algebra—the Language of Science and Mathematics" and the only formula mentioned is $I = prt$ with which many if not all of the students will be familiar, In other words, at the start the author tries to satisfy the pupil's curiosity as to what *algebra* is, and yet he connects it with something which will be recognized at once as being real and important.

The new materials which make up the greater bulk of the added pages deal with applications of mathematics in many fields especially those connected with sports and transportation—skiing, bobsledding, computing wind resistance, finding out how costly speeding is, and many others.

The pen and ink drawings that were a feature of the earlier book are used here also, but they are supplemented by numerous half tones. In the main, these are closely connected with the text, for example, an airplane view of Boston faces problems on the distance you can see from various heights—but at times the illustrations may be a source of embarrassment to the teacher. For instance, the pupils learn that formulas are needed in making a flat map from a conical projection which in turn is made from the earth's surface. Many of us would be unable to answer the inevitable "How?" On the other hand, the mention of logarithms and the pictures of a slide rule used in connection with the

laws of exponents provide an excellent opportunity to show the pupils a thing that lies not far beyond their present achievement.

In the case of any text, however, the real critic is the pupil to whom it is addressed. One of the first pupils to use the *Practical Algebra* pronounced his opinion succinctly and forcefully. He was greatly struck with the formula for "games ahead" of a base ball team and his comment was "If this is algebra, then I'm for it.'

VERA SANFORD

A Second Course in Algebra. By J. C. Stone and U. S. Mallory. Benj. H. Sanborn and Company, 1937, VII+504.

This book is a revision of an earlier edition by the same authors and consists in a general refinement throughout the book of the earlier material including some new additional material (pages 431–455) which is designed to bring the book more closely into harmony with the present recommendations of College entrance requirements.

The first six units of the book is an extensive review of the elementary algebra that is usually presented in the ninth grade, but it is reinforced by the introduction of new material rather than confined to a mere repetition of the previous material.

The book is well made and is attractively bound. Teachers of intermediate algebra will be interested to see this book prepared by two well known teachers and text-book writers.

W.D.R.

The Handmaiden of the Sciences. By Eric Temple Bell. Reynal & Hitchcock, 1937, VIII+216, Price, $2.00.

Considering the plethora of excellent popular accounts of recent developments in the various sciences, it is high time we had something of the kind in mathematical physics. And that is what we find in "The Handmaiden of the Sciences." This book is intended as a companion volume to "The Queen of the Sciences," written in 1931 and now unfortunately out of print. Whereas the older book provides us with an airplane ride over the territory of modern pure mathematics the author now takes us on a similar excursion over the field of applied mathematics. The title is perhaps a slight misnomer since the applications here discussed are almost entirely in the domain of physics.

Lucidity of exposition, liveliness, a certain humanness of treatment, and an infectious elan are the outstanding qualities of this book. The reader with little technical knowledge for whom the book is primarily intended will find here genuine meaning in such concepts as Laplace's Equation, Fermat's least or stationary principle and Riemennian manifolds. "*There are no mysterials in hyperspare as it is conceived in mathematics and in science.*" (The italics are the author's.)

An index and a bibliography, or if that be too formal, a hint here and there as to where the reader may turn to for more extended discussion might have added to the value of this altogether admirable book. Teachers of mathematics eager for a widening of horizons will find it well worth their attention.

A. M. GINSBURG

An Invitation to Mathematics. By Arnold Dresden, Henry Holt and Company, 1936, XIII ×453, Price, $2.80.

Of those who accept this "invitation" all that is demanded beyond high school algebra and plane geometry are "a readiness to concentrate and a taste for abstract thinking." The book is based on the belief that "it is not necessary to wait until the graduate school is reached before one can learn something of the more interesting and broadly significant parts of mathematics." It comprises a broad and coherent view of the theory of numbers, modern geometry, analysis, projective geometry and has even a glimpse of topology. What a brave departure from the course in "general mathematics" usually encountered in the first or second years of college!

With the familiar Pythagorean theorem and the related problem of finding the sets of relatively prime integers which form the solution of the equation $p^2+q^2=r^2$ as our springboard, we are immediately but logically plunged into the theory of infinite sets. Later in the book the quest for primitive Pythagorean triples, a beautiful illustration, by the way, of the methods of mathematical reasoning, leads just as naturally to a consideration of "Fermat's last theorem" and other topics in the theory of numbers.

If any apology for the inclusion of number theory in the book were needed, the author could hardly have found a more eloquent one than that contained in an address by Professor Hardy and referred to in the book. "The elementary theory of numbers should be one of the very best subjects for early mathematical instruction. It demands very little previous knowledge, its subject matter is tangible and familiar; the processes of reasoning which it employs are simple, general and few and it is unique among the mathematical sciences in its appeal to natural human curiosity. A month's intelligent instruction in the theory of numbers ought to be twice as instructive, twice as useful and at least ten times as entertaining as the same amount of 'calculus for engineer's.'"

For the term "imaginaries" Professor Dresden proposes the name "normal numbers" (since they are represented on an axis perpendicular to the axis of reals) "in the hope that it will divest these perfectly innocent numbers of the awe-inspiring mysteriousness which has always clung to them." But if we are going to have innovations in nomenclature why retain "natural numbers" or "real numbers" which are no whit more or less natural or real than any other types of numbers found in mathematics?

The problems to be done by the student form an integral part of the book. Solving them should prove at once a challenge and a pleasure for in these problems the true nature of mathematics is not submerged in a sea of manipulative techniques.

Those who prefer their mathematics "straight" may find the tone of the book at times too gay, lyrical, even rhapsodic. Others for whom the science is not untouched with glamor may see in the "Invitation to Mathematics" something much more than a text-book.

A. M. GINSBURG

THE BOARD OF DIRECTORS OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
PALMER HOUSE, CHICAGO, ILL., FEBRUARY 20, 1937

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